Black-body luminance and magnitudes per square arcsecond in the Johnson-Cousins BVR photometric bands

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Abstract—A relevant number of light pollution studies deal with unwanted visual effects of artificial light at night, including the anthropogenic luminance of the sky that hinders the observation of celestial bodies which are the main target of ground-based astrophysical research, and a key asset of the intangible heritage of humankind. Most quantitative measurements and numerical models, however, evaluate the anthropogenic sky radiance in any of the standard Johnson-Cousins UBVRI photometric bands, generally in the V one. Since the Johnson-Cousins V band is not identical with the visual CIE V(λ) used to assess luminance, the conversion between these two photometric systems turns out to be spectrum-dependent. Given its interest for practical applications, in this Letter we provide the framework to perform this conversion and the transformation constants for black-body spectra of different absolute temperatures.

One of the side-effects of the use of artificial light at night is the increase of night sky brightness above its expected natural levels [1], due to the scattering of upward light emissions off the molecular and aerosol constituents of the atmosphere [2]. The calculation and measurement of the integrated spectral sky radiance are very often made in the widely used astrophysical Johnson-Cousins V photometric band [3], and its visual effects, in terms of luminance, are loosely identified with the ones that would be obtained if the correct visual spectral sensitivity band V(λ) [4] were used instead.

However, since the Johnson-Cousins (JC) V and the CIE V(λ) bands are not coincident, the relationship between the integrated radiance in both of them turns out to be dependent on the spectrum of the incident light [5-6]. The same applies to the blue (B) and red (R) JC bands (Fig. 1). Given the widespread use of JC V band integrated radiances to deduce approximate estimates of visual percepts, the purpose of this Letter is to provide an accurate framework for the transformation between these two systems, and its application to black-body spectra of different absolute temperature. Explicit constants for the transformations from B and R bands integrated radiances are also provided in graphical form. The U (ultraviolet) and I (near infrared) bands of the UBVRI JC photometric system are of lesser interest for this particular application and will not be included here, although the corresponding transformation constants can be straightforwardly obtained following the steps outlined below.

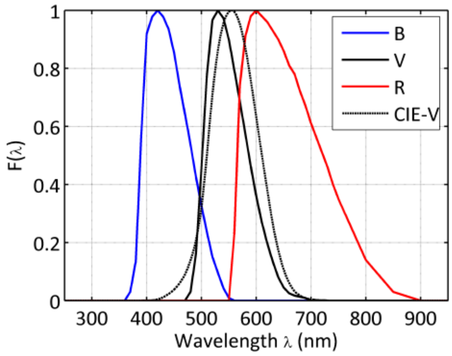

Fig. 1. CIE V(λ) and Johnson-Cousins B, V, and R bands.

Let us remind that the luminance L (units cd/m^2) produced by a spectral radiance distribution $L_r(\lambda)$ (W·m^{-2} sr^{-1}·nm^{-1}) for photopically adapted observers is obtained by weighting it by the CIE V(λ) function and integrating across wavelengths:

$$L = k_m \int V(\lambda) L_r(\lambda)\,\mathrm{d}\lambda \,, \tag{1}$$

where k_m=683 lm/W is the scaling factor relating luminance to radiance for monochromatic radiation at wavelength λ_0=555 nm, corresponding to the maximum of the V(λ) function.

The night sky brightness corresponding to the same $L_r(\lambda)$ spectral radiance, in turn, is customarily expressed in astrophysics in the non-SI, negative logarithmic scale of units called "magnitudes per square arcsecond", which for any generic JC filter band, $F(\lambda)$, with $F \in \{B, V, R\}$ are defined as:

$$m_F = -2.5 \log_{10} \frac{\int F(\lambda) L_r(\lambda)\,\mathrm{d}\lambda}{\int F(\lambda) L_{r0}(\lambda)\,\mathrm{d}\lambda} \,, \tag{2}$$

where $L_{r0}(\lambda)$ is the arbitrary, but clearly specified, spectral radiance distribution that is chosen to set the origin of the magnitudes scale.

From Eqs. (1) and (2) one immediately obtains:

$$L = L_0 \times 10^{-0.4 m_F} \,, \tag{3}$$

where the "zero-point" radiance L_0 (cd/m^2) is given by

$$L_0 = k_m \left[\frac{\int V(\lambda) L_r(\lambda) \mathrm{d}\lambda}{\int F(\lambda) L_r(\lambda) \mathrm{d}\lambda} \int F(\lambda) L_{r0}(\lambda) \mathrm{d}\lambda \right] \tag{4}$$

Note that an important and frequently overlooked factor for the correct interpretation of the meaning of m_T and of the conversion Eqs. (3) and (4), is the particular choice of radiance distribution taken as reference, $L_{r0}(\lambda)$. The scale of astronomical magnitudes - henceforth denoted by μ_T and not to be confounded with the magnitudes per square arcsecond, m_T, see below- corresponds to band integrated irradiances E ($\mathrm{W \cdot m^{-2}}$) and its zero point has been defined differently across history, using e.g. sun-based or Vega (α Lyr) spectral irradiance distributions [7]. A more convenient choice for our present purposes, not tied to the experimental determination of the spectral irradiance produced by any particular star, is the absolute (AB) magnitude scale, whose reference spectral irradiance $E_{r0}(\nu)$ is constant when expressed in the frequency (Hz) domain, and is implicitly defined by [8]:

$$\mu_{AB} = -2.5 \log_{10} E_r(\nu) - 48.60 \,, \tag{4}$$

with the measured irradiances $E_r(\nu)$ expressed in $\mathrm{ergs \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}}$. Note that in Ref. [9] this equation has a misprint in the sign of the -48.60 zero-point term. From Eq. (4), the AB zero-point reference spectral irradiance distribution in the frequency domain $E_{r0}(\nu)$ (i.e., $E_r(\nu)$ for $\mu_{AB}=0$) is, with four significant digits,

$$E_{r0}(\nu) = 10^{-0.4 \times 48.60} = 3631 \times 10^{-20} \ \mathrm{ergs \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}}, \tag{5}$$

or, equivalently, $E_{r0}(\nu)$=3631 Jy, where 1 Jy (jansky) = 10^{-26} $\mathrm{W \cdot m^{-2} \cdot Hz^{-1}}$. The spectral irradiance in the frequency domain can be expressed in the wavelength domain taking into account that $\nu=c/\lambda$, where $c=2.99792458 \times 10^8$ $\mathrm{m \cdot s^{-1}}$ is the speed of light, and that $E_{r0}(\nu)\mathrm{d}\nu = E_{r0}(\lambda)\mathrm{d}\lambda$. Since $|\mathrm{d}\nu/\mathrm{d}\lambda|=c/\lambda^2$, it follows that $E_{r0}(\lambda)=3631 \times 10^{-26} \times c/\lambda^2$ $\mathrm{W \cdot m^{-2} \cdot m^{-1}}$, with c expressed in $\mathrm{m \cdot s^{-1}}$ and λ in m.

The corresponding zero-point radiance, $L_{r0}(\lambda)$, is defined as the spectral radiance that would give rise to the zero-point irradiance, $E_{r0}(\lambda)$, under normal incidence, if the source would subtend a solid angle $\Delta\omega$=1 square arcsecond as seen from the observer. Since 1 arcsec^2= 2.3504×10^{-11} sr, we finally have, for the AB zero-point spectral radiance, in $\mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot m^{-1}}$,

$$L_{r0}(\lambda) = \frac{E_{r0}(\lambda)}{\Delta\omega} = \frac{3631 \times 10^{-26}}{2.3504 \times 10^{-11}} \times \frac{c\,[\mathrm{m\ s^{-1}}]}{\lambda^2\,[\mathrm{m}]} . \tag{6}$$

With this AB reference radiance we can calculate the zero-point luminance in Eq. (4) for any particular spectral radiance distribution whose JC magnitudes per square arcsecond are given by m_F in Eq. (3). A particular subset of spectra of practical interest are the black-body ones. Their spectral radiance is given by

$$L_r(\lambda) = \frac{2hc^2}{\lambda^5} \times \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \,, \tag{7}$$

where T is the absolute temperature (in kelvin, K), $h=6.62607015 \times 10^{-34}$ J·s is the Planck constant, and $k=1.380649 \times 10^{-23}$ $\mathrm{J \cdot K^{-1}}$ is the Boltzmann constant, according to the 2019 redefinition of SI base units.

The results for L_0 (cd/m^2) of the B, V and R bands are displayed in Fig. 2. The curve corresponding to the V band, of particular practical interest, is shown in Fig. 3.

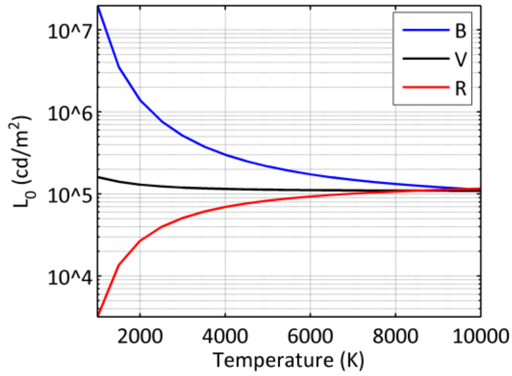

Fig. 2. Zero-point luminances for black-body source radiation of different temperatures, corresponding to the AB magnitude per square arcsecond system for the Johnson-Cousins B, V, and R bands.

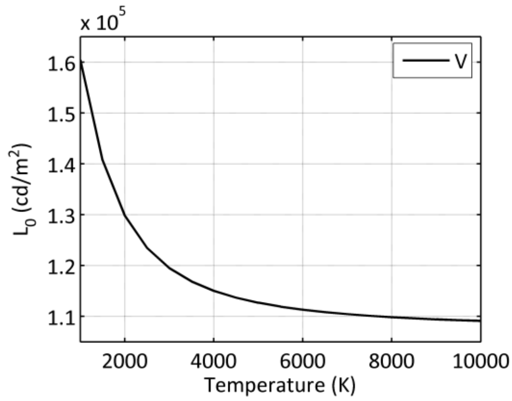

Fig. 3. Detailed view of the zero-point luminances for black-body source radiation of different temperatures, corresponding to the AB magnitude per square arcsecond system for the Johnson-Cousins V band.

Given the widespread use of the JC V-magnitudes to estimate luminances, the numerical values of L_0 at T intervals of 500 K are specified in the second column of Table 1.

Table 1. Zero-point luminances L_0 for black-body source radiation of different temperatures, corresponding to the AB magnitude per square arcsecond system for the Johnson-Cousins V band, and modified zero-point luminances L_0' for a Johnson-Cousins V system in which the magnitude of Vega (α Lyr) be identically equal to 0.00.

Black body temperature T (K)	L_0 (in units 10^5 cd/m^2)	L_0' (in units 10^5 cd/m^2)
1000	1.6044	1.5751
1500	1.4071	1.3814
2000	1.2979	1.2742
2500	1.2345	1.2120
3000	1.1949	1.1731
3500	1.1686	1.1473
4000	1.1502	1.1292
4500	1.1369	1.1161
5000	1.1269	1.1063
5500	1.1192	1.0987
6000	1.1131	1.0928
6500	1.1083	1.0881
7000	1.1044	1.0842
7500	1.1012	1.0811
8000	1.0985	1.0784
8500	1.0962	1.0762
9000	1.0943	1.0743
9500	1.0926	1.0727
10000	1.0912	1.0712

The values of L_0 for the V band in Fig. 3 and Table 1 can be directly used to calculate the luminance L of the black-body radiation whose AB V-band magnitudes per square arcsecond are $m_{V,AB}$, as

$$L = L_0 \times 10^{-0.4 m_{V,AB}}, \tag{8}$$

see Eq. (3).

As a final remark, if instead of the AB spectral radiance given in Eq. (6) the spectral radiance of the star Vega (α Lyr) were taken as the reference for setting the zero-point of the V-magnitude scale, the magnitudes per square arcsecond in both systems would be related by [10]

$$m_{V,AB} - m_{V,Vega} = 0.02 \tag{9}$$

Hence, using Vega as the reference, the luminances of the black-body sources in terms of V-magnitudes per square arcsecond are given by

$$L = L_0 \times 10^{-0.4\times 0.02} \times 10^{-0.4 m_{V,Vega}} = L_0' \times 10^{-0.4 m_{V,Vega}}. \tag{10}$$

The values of the zero-point luminances in this shifted magnitude scale, L_0', are given in the third column of Table 1.

The above results show that the most commonly used transformation between V-magnitudes per square arcsecond and luminances, which is based on a generic zero-point luminance of 1.08×10^{-5} cd/m^2 has not universal validity, even if restricted to the measurement of radiation from black-body sources. For this last type of spectra, especially for moderate to low color temperatures, the values provided in Table 1 should be used instead.

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References

[1] F. Falchi *et al.*, Sci. Adv. **2**, e1600377 (2016). doi:10.1126/sciadv.1600377

[2] M. Kocifaj, J. Quantitative Spectroscopy Radiative Transfer **181**, 2 (2016).

[3] M.S. Bessel, Publications of the Astronomical Society of the Pacific, **102**, 1181 (1990).

[4] CIE, Commision Internationale de l'Éclairage. CIE 1988 2° *Spectral Luminous Efficiency Function for Photopic Vision* (Vienna, Bureau Central de la CIE, 1990).

[5] S. Bará, Intern. J. Sustainable Lighting IJSL **19**(2), 104 (2017). doi: 10.26607/ijsl.v19i2.77

[6] A. Sánchez de Miguel, M. Aubé, J. Zamorano, M. Kocifaj, J. Roby, C. Tapia. Monthly Notices of the Royal Astronomical Society **467**(3), 2966 (2017). doi:10.1093/mnras/stx145

[7] M.S. Bessell, Annual Reviews of Astronomy and Astrophysics, **43**, 293 (2005).

[8] J.B. Oke, The Astrophysical J. Suppl. Series **236**(27), 21 (1974).

[9] J.B. Oke, J.E. Gunn, The Astrophysical J. **266**, 713 (1983).

[10] M.R. Blanton, S. Roweis S., The Astronomical J. **133**(2), 734(2007).

