Convolution Algorithm for Numerical Reconstruction of Digitally Recorded Holograms with an Extended Field of View

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Received November 29, 2010; accepted December 13, 2010; published December 31, 2010

Abstract— In this letter, the range of application of the convolution approach to numerical reconstruction of digitally recorded holograms is extended. By numerical manipulation of the digitally recorded hologram, the convolution approach is used for reconstructing digital holograms of objects larger than the recording device. Experimental results are shown to validate the proposed method.

Numerical reconstruction of digital recorded holograms, named digital holography (DH), is supported on the same foundations as of optical holography; it can be modelled as a two-diffraction-step technique. In the first step, an object located at a plane z=0 scatters the impinging coherent wave; the scattered optical field interferes with a reference wave on the so called hologram plane placed at a distance $z=d$. A 2D-discrete detector (CCD or CMOS camera) is placed at this plane, so that a sampled version of the interference pattern $I(x_h,y_h)$ is recorded and transferred to a computer for its further processing. The recorded intensity carries on information about i) the intensities of the reference and scattered waves, and ii) the interference term between the reference and the scattered wave [1]. In the second step, the 3D-information of the object is retrieved by diffracting the conjugated reference wave as it impinges on $I(x_h, y_h)$. This diffraction process, known as hologram reconstruction, can be carried out by evaluating the Fresnel-Kirchhoff diffraction formula [2]:

$$E\left(x_i,y_i,z\right)=-\frac{i}{2\lambda}\frac{E_0\exp\left(ikr_0\right)}{r_0}\iint_{\Sigma} I\left(x_h,y_h\right)r^*\left(x_h,y_h\right)\frac{\exp\left(iks\right)}{s}\left[1+\cos\chi\right]dx_h dy_h \quad . \quad (1)$$

In Eq. (1) we have considered that the reference wave has wavelength λ, amplitude E_0, spatial distribution $r^*(x_h, y_h)$ and r_0 wavefront radius; the hologram extends over an area $\Sigma, i=\sqrt{-1}$ and $k=2\pi\lambda^{-1}$ is the wave number. The $\left[1+\cos\chi\right]$ term is the inclination factor, such that $\chi\to 0$ for small numerical aperture applications. The distance between any point on the hologram to each point on the reconstruction plane is given by:

$$s=\sqrt{z^2+\left(x_h-x_i\right)^2+\left(y_h-y_i\right)^2}\ . \quad (2)$$

In many practical applications the reference wave is a homogeneous plane wave impinging perpendicularly to the hologram plane, such that it can be represented as a constant field of amplitude E_0. This condition and small numerical aperture systems will be considered in the text that follows.

Different approaches to compute Eq. (1) are found in the literature; Fresnel approach relies on the parabolic approximation of the distance

$$s\approx z\left[1+\frac{1}{2}\left(\frac{x_h-x_i}{z}\right)^2+\frac{1}{2}\left(\frac{y_h-y_i}{z}\right)^2\right]$$

for the exp(iks) term and $s\approx z$ in the denominator. These approximations transform Eq. (1) into:

$$E\left(x_i,y_i,z\right)=\frac{E_0\exp\left(ikz\right)}{i\lambda z}\exp\left[\frac{i\pi}{\lambda z}\left(x_i^2+y_i^2\right)\right]\iint_{\Sigma}\left\{I\left(x_h,y_h\right)\exp\left[\frac{i\pi}{\lambda z}\left(x_h^2+y_h^2\right)\right]\exp\left[-\frac{i2\pi}{\lambda z}\left(x_h x_i+y_h y_i\right)\right]\right\}dx_h dy_h \quad . \quad (3)$$

Equation (3) can be numerically calculated by discrete Fourier transforming the product of the recorded hologram $I(x_h,y_h)$ times the Fresnel phase at the hologram plane $\exp\left[\frac{i\pi}{\lambda z}\left(x_h^2+y_h^2\right)\right]$. When the hologram is recorded on N_x, N_y pixels, the sizes of the reconstructed $\Delta x_i,\Delta y_i$ and recorded $\Delta x_h,\Delta y_h$ pixels are related by

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$\Delta x_i = \lambda z / N_x \Delta x_h, \Delta y_i = \lambda z / N_y \Delta y_h$, leading to a total reconstructed area of $x_i y_i = (\lambda z)^2 / \Delta x_h \Delta y_h$.

Another approach to evaluate Eq. (1) relies on writing such an equation as a superposition integral:

$$E(x_i, y_i, z) = \iint h(x_i, y_i; x_h, y_h) I(x_h, y_h) dx_h dy_h \quad (4)$$

where $h(x_i, y_i; x_h, y_h)$ is the impulse response of the free space given by:

$$h(x_i, y_i; x_h, y_h) = \frac{E_0}{i\lambda} \frac{\exp(iks)}{s} = \frac{E_0}{i\lambda} \frac{\exp\left(ik\sqrt{z^2 + (x_h - x_i)^2 + (y_h - y_i)^2}\right)}{\sqrt{z^2 + (x_h - x_i)^2 + (y_h - y_i)^2}}. \quad (5)$$

The evaluation of Eq. (4) is carried out by accounting the convolution property of Fourier transforms [3]: i) $h(x_i, y_i; x_h, y_h)$ and $I(x_h, y_h)$ are independently Fourier transformed; ii) These Fourier transforms are pixel-wise multiplied; iii) $E(x_i, y_i, z)$ equals the inverse Fourier transform of the latter product. It is worth noting that the Fresnel approach produces the scattered field $E(x_i, y_i, z)$ on the spatial frequency space, while the convolution approach produces it on the spatial domain. For this reason the pixel size of the reconstructed scattered wave equals that of the recorded hologram. Therefore, the total reconstructed area will be given by $x_i y_i = N_x \Delta x_h N_y \Delta y_h$.

Different methods to reconstruct digital holograms have found their range of application based on careful choice of the reconstruction distance. The aim of this choice is to avoid fast changes of phase which would ruin the reconstruction process. Another factor to consider on the choice of reconstruction algorithms is the size of the reconstructed image field. In this matter, the Fresnel approach has fewer constraints than the convolution method. The latter suffers from the restriction that the largest area of the reconstructed image equals that of the recording device.

Figure 1 shows the reconstruction of one hologram done by the Fresnel approach (panel A) and by the convolution one (panel B). Spatial filtering on the Fourier domain has been applied for all the reconstruction presented in this work. With this well known procedure, the inconvenient effects of the twin image and the zeroth-diffraction-order have been removed from the reconstructed images.

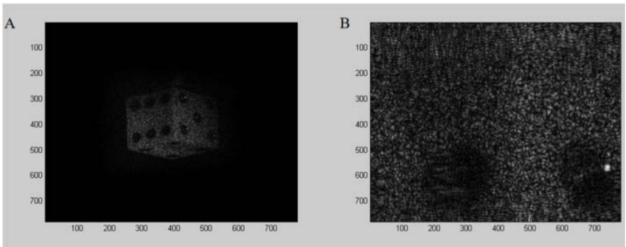

Fig. 1. Reconstruction of one hologram with the Fresnel approach (A) and the convolution method (B).

The hologram was recorded on a CCD camera with 780x780 square pixels of 11μm side. The illumination wavelength was 633nm and the object was placed at a distance of 1.05m from the camera. Since the hologram is reconstructed with the Fresnel approach, these parameters lead to a reconstructed pixel size of 77.4μm and a reconstructed area of 60.4x60.4mm^2. As the reconstruction is done with the convolution approach the reconstructed pixel size equals 11μm and 8.58x8.58mm^2 is the whole area of reconstruction. Due to a limited reconstructed area of the convolution approach, it is not possible to have a full view of the die that is seen in panel A. However, the smaller pixel size of the reconstructed image for the convolution approach could lead to a better resolved image if the reduced field of view is somehow increased.

The field of view of the convolution approach can be enlarged by increasing the number of pixels of the input hologram. Since the number of pixels of the CCD camera is factory set, zero padding of the hologram can help to increase such a field of view [4]. In Fig. 2, the hologram reconstructed in Fig. 1 has been padded to 2048x2048pixels and thereafter, for fair comparison, reconstructed by the two mentioned approaches.

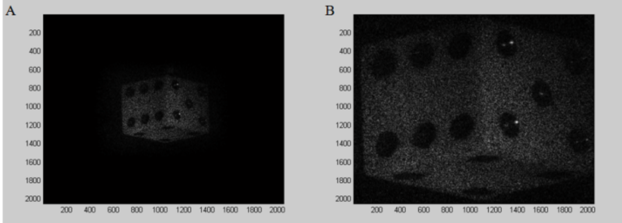

Fig. 2. Reconstruction of the same hologram as in Fig. 1 after padded to 2048x2048 pixels; Fresnel approach (A) and convolution method (B).

In both panels of Fig. 2 a full image of the die is reproduced, but the 4 megapixels of the reconstructed image are used on different ways. In panel A, the Fresnel approach averages the reconstructed image and the die itself is restricted to a small area of the whole image. The new pixel size of the reconstructed image is reduced to $\Delta x_i = \Delta y_i = 29.5\,\mu\text{m}$; this means a reduction of the speckle size while keeping the same field of view, leading to an apparent increase of resolution. The convolution approach, panel B, shows an enlargement of the field of view that fits the whole image of the die. Since the pixel size is kept fixed, the convolution approach could be understood as a zoom effect of the Fresnel reconstructed image without the need of any digital average or interpolation, while keeping the speckle size given by the pixel size of the recording device. One could be tempted to claim an increase in the lateral resolution with the reconstruction by the convolution approach; however, as in any other imaging system, the resolution is given by the size of the aperture (CCD camera in this case), wavelength and observation distance. Since these parameters are the same in both reconstruction methods, the lateral resolution is the same and only the zoom effect takes place.

To better illustrate this later idea, Fresnel and convolution approaches have been applied to reconstruct holograms of a USAF 1951 resolution test target. The holograms were recorded at a distance of 40cm with a CCD camera with 640x480 pixels and pixels of $7.4\text{x}7.2\mu\text{m}^2$; the target, 1.5cm side, was illuminated with a coherent light of the 633nm wavelength. These parameters mean that in order to reconstruct the whole target with the convolution approach one must pad the original holograms to at least 2048x2048 pixels.

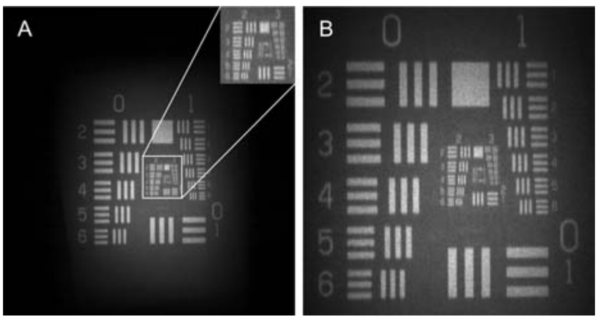

Fig. 3. Reconstruction of a USAF 1951 test target; Fresnel approach (A) and convolution method (B). The speckle noise has been reduced by using the uncorrelated superposition of holograms [5].

When considering the whole field of view and compared panels A and B of Fig. 3, it is apparent that there is an increase of lateral resolution by reconstructing with the convolution approach, see panel B. However, by a closer look at the inset in panel A, where a digital zoom of the region of interest is shown, one can see the same order of resolution as that in panel B.

The advantage of doing the reconstruction of the digital holograms via the convolution approach, lies in the fact that this methodology produces reconstructions that resemble the results of a digital zoom performed in the reconstruction made by other approaches; for this reason the main field of application of the convolution methodology is the reconstruction of holograms of a field of particles or in-line holograms of microscopic objects [6].

In summary, among different options of numerical reconstruction of the optical field from digitally recorded holograms, Fresnel's and convolution approaches are the ones used. The features of each are the key point in determining which method to employ for particular experiments. In this letter, the apparent limitation of the convolution approach meant to reconstruct only objects with dimensions equal or smaller than those of the recording device is used to produce a zoom effect of the reconstructed field. To avoid the constraint of the reconstructed size, a simple approach of zero padding is used and objects larger than the dimensions of the recording device are successfully reconstructed.

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