Coherence matrix of plasmonic beams

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Abstract—We consider monochromatic electromagnetic beams of surface plasmon-polaritons created at interfaces between dielectric media and metals. We theoretically study non-coherent superpositions of elementary surface waves and discuss their spectral degree of polarization, Stokes parameters, and the form of the spectral coherence matrix. We compare the polarization properties of the surface plasmons-polaritons as three-dimensional and two-dimensional fields concluding that the latter is superior.

Physics of electromagnetic beams has gained considerable attention owing to recent advances in the study of the Airy beams, Bessel beams, Bessel-Gauss beams, etc.[1]-[4]. Any of these beams can be formed using coherent superposition of elementary waves, e.g., plane waves. When elementary waves are not coherent, the beams are characterized by the Stokes parameters and the degree of polarization. These parameters can be conveniently combined in the coherence matrix [5]. If an electromagnetic field has no direction of propagation, i.e. it is not a beam, the elementary waves propagate in various random directions in the three-dimensional space. Such three-dimensional fields (like thermal fields) are described by the generalized coherence matrix and degree of polarization [6]-[8]. These fields exactly are assumed to be excitation sources for the surface waves in the current investigation.

Localized surface plasmon-polariton (SPP) beams are not very special objects. Similarly to optical beams, they can be generated as Bessel and Airy plasmonic beams [11], [12]. That is why we can expect that the non-coherent superposition of the surface waves should be put forth. In the previous works [9], [10] the spatial coherence and polarization properties of the SPP fields were discussed for some specific cases.

We use the tensorial generalization of the coherence matrix for statistically stationary fields, which is called a light beam tensor Φ [13]-[15]. Hermitian conjugated tensor Φ ($\Phi^+ = \Phi$) in the coordinate-free notations, which ignore setting a specific coordinate system, is:

$$\Phi(\omega, \mathbf{r}) = \sum_s \mathbf{E}^{(s)} \otimes \mathbf{E}^{(s)*}, \tag{1}$$

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where dyad $\mathbf{a} \otimes \mathbf{b}$ is the direct (Kronecker) product of vectors **a** and **b**, $\mathbf{E}^{(s)}$ is the electric field of an elementary wave, the asterisk denotes complex conjugate. When the elementary non-coherent plane waves propagate in the direction of unit vector **n**, the orthogonal condition $\mathbf{E}^{(s)}\mathbf{n}=0$ brings us to the two-dimensional beam tensor Φ restricted by $\Phi\mathbf{n}=\mathbf{n}\Phi=0$, where the contraction of tensor Φ with vector **n** is assumed. Then the tensor has two non-zero eigenvalues λ_1 and λ_2 and can be presented as

$$\Phi_2 = (\lambda_1 - \lambda_2)\mathbf{u}_1 \otimes \mathbf{u}_1^* + \lambda_2 \mathbf{I}, \tag{2}$$

where $\mathbf{I} = \mathbf{u}_1 \otimes \mathbf{u}_1 + \mathbf{u}_2 \otimes \mathbf{u}_2 = \mathbf{1} - \mathbf{n} \otimes \mathbf{n}$ is the projector onto the plane perpendicular to vector **n**, $\mathbf{u}_1$ and $\mathbf{u}_2$ are the in-plane normalized eigenvectors such that $|\mathbf{u}_1|=|\mathbf{u}_2|=1$. The first and second terms in the right-hand side of equation (2) correspond to the completely polarized and unpolarized parts, respectively.

In general, the field has no definite propagation direction and can be treated as three-dimensional one. It is sufficient for our purposes to consider light as a superposition of completely polarized light with the three-dimensional complex vector $\mathbf{u}_1$ and three-dimensionally unpolarized field proportional to the 3D identity tensor **1** (more general description is presented in Refs. [6]-[8]). Thus one introduces

$$\Phi_3 = (\lambda_1 - \lambda_2)\mathbf{u}_1 \otimes \mathbf{u}_1^* + \lambda_2 \mathbf{1}, \tag{3}$$

Let us move to the SPPs now. When the boundary conditions for a plane wave incident onto the metal-dielectric interface are satisfied, there appear electron oscillations in the metal and a surface electromagnetic wave coupled to these collective oscillations (Fig. 1). The wave is called SPP [16]. It is well localized at the interface and can be described in the dielectric medium as

$$\mathbf{E}_d = e^{ik_p\mathbf{mR} - \kappa_d z - i\omega t} \frac{A}{\varepsilon_d}\left(i\frac{\kappa_d}{k_0}\frac{\mathbf{k}_p}{k_p} - \frac{k_p}{k_0}\mathbf{e}_z \right),$$
$$\mathbf{H}_d = e^{ik_p\mathbf{R} - \kappa_d z - i\omega t} A \frac{\mathbf{e}_z \times \mathbf{k}_p}{k_p}, \tag{4}$$

where A is the complex amplitude of the wave, $\mathbf{R}$ is the radius-vector at the metal-dielectric interface, $\mathbf{e}_z$ is the unit vector normal to the interface, $k_0=\omega/c$ is the wavenumber in vacuum, $k_p = k_0\sqrt{\varepsilon_d\varepsilon_m/(\varepsilon_d+\varepsilon_m)}$ is the wavenumber of a surface wave, $\kappa_d = \sqrt{k_p^2 - k_0^2\varepsilon_d}$, ε_d and ε_m are complex dielectric permittivities of the dielectric material and metal, $\mathbf{k}_p=k_p\mathbf{m}$, and $\mathbf{m}$ is a real vector defining the direction of propagation of an elementary wave. If one knows the wavevectors of the elementary waves (e.g., all plane waves propagate along vector $\mathbf{n}$), the surface waves are completely polarized even for the unpolarized excitation source: only TM-modes with the same wavenumbers $\mathbf{k}_p$ are possible.

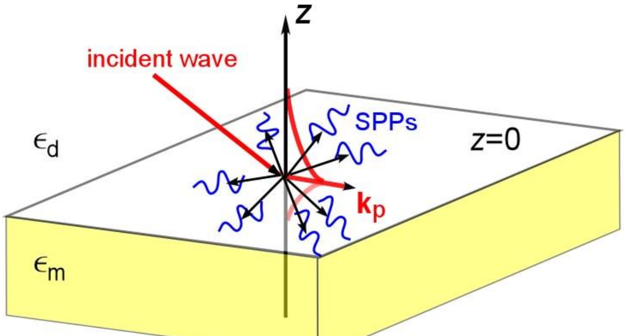

Fig. 1. Non-coherent SPPs excited in random directions at the metal-dielectric interface z=0.

The situation changes when the elementary waves of the excitation field propagate in random directions. Then the surface wavenumbers $\mathbf{k}_p^{(s)}=k_p\mathbf{m}^{(s)}=k_p(\cos\psi^{(s)}\mathbf{e}_x+\sin\psi^{(s)}\mathbf{e}_y)$ differ for distinct elementary waves. Spectral light beam tensor is of the form

$$\Phi_3^{pl} = \frac{e^{-2\kappa_d z}}{|\varepsilon_d|^2}\sum_s e^{-2\,\mathrm{Im}(k_p)\mathbf{m}^{(s)}\mathbf{R}}\,|A^{(s)}|^2\,\mathbf{b}^{(s)}\otimes\mathbf{b}^{(s)*},\qquad (5)$$
$$\mathbf{b}^{(s)} = i\frac{\kappa_d}{k_0}\mathbf{m}_p^{(s)} - \frac{k_p}{k_0}\mathbf{e}_z.$$

Tensor Φ_3^{pl} is not arbitrary as additional restrictions are imposed on the components. The beam tensor at the metal-dielectric interface (x, y)

$$\Phi_2^{pl} = \mathbf{I}_z\Phi_3^{pl}\big|_{z=0}\mathbf{I}_z = \frac{|\kappa_d|^2}{|\varepsilon_d|^2k_0^2}\sum_s e^{-2\,\mathrm{Im}(k_p)\mathbf{m}^{(s)}\mathbf{R}}\,|A^{(s)}|^2\,\mathbf{m}^{(s)}\otimes\mathbf{m}^{(s)}\qquad (6)$$

is obviously real-valued, i.e. $\Phi_2^{pl} = (\Phi_2^{pl})^*$. Here $\mathbf{I}_z = \mathbf{1} - \mathbf{e}_z\otimes\mathbf{e}_z$ is the projector onto the metal-dielectric interface. The non-diagonal components are imaginary, that is $(\mathbf{e}_z\Phi_3^{pl}\mathbf{e}_{x,y}) = -(\mathbf{e}_z\Phi_3^{pl}\mathbf{e}_{x,y})^*$. Then the spectral beam tensor (3) for the plasmon-polaritons takes the form

$$\Phi_3^{pl} = (\lambda_1-\lambda_2)(\mathbf{a}+i\alpha\mathbf{e}_z)\otimes(\mathbf{a}-i\alpha\mathbf{e}_z)+\lambda_2\mathbf{1},\qquad (7)$$

where $\mathbf{a}=a_x\mathbf{e}_x+a_y\mathbf{e}_y$ and α are real quantities. Complex vector $\mathbf{a}+i\alpha\mathbf{e}_z$ is the eigenvector of Φ_3^{pl} normalized by the unity, thus $\mathbf{a}^2+\alpha^2=1$. The three-dimensional plasmonic beam tensor (7) is fully defined by four independent real-valued parameters: λ_1, λ_2, a_x, and a_y. Therefore, the polarization of the plasmonic beam is defined by four Stokes parameters:

$$S_0 = (\Phi_3^{pl})_{xx} + (\Phi_3^{pl})_{yy} + (\Phi_3^{pl})_{zz} = \lambda_1 + 2\lambda_2,\qquad (8a)$$

$$S_1 = (\Phi_3^{pl})_{xx} - (\Phi_3^{pl})_{yy} = (\lambda_1-\lambda_2)(a_x^2 - a_y^2),\qquad (8b)$$

$$S_2 = (\Phi_3^{pl})_{xy} + (\Phi_3^{pl})_{yx} = 2(\lambda_1-\lambda_2)a_xa_y,\qquad (8c)$$

$$S_3 = i\left[(\Phi_3^{pl})_{zx} - (\Phi_3^{pl})_{xz}\right] = -2(\lambda_1-\lambda_2)\alpha.\qquad (8d)$$

They look as conventional Stokes parameters, but S_3 is out of the metal-dielectric interface (x, y). The three-dimensional spectral degree of polarization is the ratio of the intensity of completely polarized light and the whole intensity of the localized field. For the three-dimensional field, the degree of polarization takes the form [6]-[8]

$$P_3 = \sqrt{\frac{3Tr((\Phi_3^{pl})^2)}{2\left(Tr(\Phi_3^{pl})\right)^2} - \frac{1}{2}},\qquad (9)$$

where the light beam tensor invariants equal

$$Tr(\Phi_3^{pl}) \propto \sum_s e^{-2\,\mathrm{Im}(k_p)\mathbf{m}^{(s)}\mathbf{R}}\,|A^{(s)}|^2|\mathbf{b}^{(s)}|^2,$$
$$Tr((\Phi_3^{pl})^2) \propto \sum_{s,s'} e^{-2\,\mathrm{Im}(k_p)(\mathbf{m}^{(s)}+\mathbf{m}^{(s')})\mathbf{R}}\,|A^{(s)}A^{(s')}|^2|\mathbf{b}^{(s)*}\mathbf{b}^{(s')}|^2.$$

Treating SPPs as three-dimensional fields can be excessive, because the z-components in vectors $\mathbf{b}^{(s)}$ are not fluctuating. Therefore, the wave field can be unambiguously reduced to the two-dimensional field with either electric or magnetic field at the metal-dielectric surface (x, y). Non-coherent SPP field has no specific direction, hence, electric (magnetic) field is not directed along the x (y) axis, but covers various directions in the metal-dielectric plane. Moreover, in this case the spectral beam tensor of the two-dimensional field depends on the shape of the interface (planar, cylindrical, or spherical), what is quite natural. The two-dimensional plasmonic beam tensor (6) for the planar interface is real and, therefore, has the form

$$\Phi_2^{pl} = (\lambda_1 - \lambda_2)\mathbf{a} \otimes \mathbf{a} + \lambda_2 \mathbf{I}_z, \qquad (10)$$

where $\mathbf{a}\mathbf{I}_z$=$\mathbf{I}_z\mathbf{a}$=$\mathbf{a}$=$a_x\mathbf{e}_x+a_y\mathbf{e}_y$, $\mathbf{a}^2$=1. There are only 3 independent quantities in equation (10): λ_1, λ_2, and a_x. Thus, one can introduce three Stokes parameters in the plane (x, y):

$$\begin{aligned} S_0 &= (\Phi_2^{pl})_{xx} + (\Phi_2^{pl})_{yy} = \lambda_1 + \lambda_2, \\ S_1 &= (\Phi_2^{pl})_{xx} - (\Phi_2^{pl})_{yy} = (\lambda_1 - \lambda_2)(2a_x^2 - 1), \\ S_2 &= (\Phi_2^{pl})_{xy} + (\Phi_2^{pl})_{yx} = 2(\lambda_1 - \lambda_2)a_x\sqrt{1-a_x^2}. \end{aligned} \qquad (11)$$

The forth Stokes parameter S_3 (spin angular momentum) is identically equal to zero for elementary SPP waves, because the electric fields are radially directed (along different radii $\mathbf{m}^{(s)}$).

The spectral degree of polarization of the two-dimensional field is calculated using the well-known relationship $P_2 = \sqrt{2Tr((\Phi_2^{pl})^2)/(Tr(\Phi_2^{pl}))^2 - 1}$, where the invariants of the coherence matrix (6) are determined by

$$Tr(\Phi_2^{pl}) \propto \sum_s e^{-2\,\mathrm{Im}(k_p)\mathbf{m}^{(s)}\mathbf{R}}\,|A^{(s)}|^2,$$

$$Tr((\Phi_2^{pl})^2) \propto \sum_s e^{-2\,\mathrm{Im}(k_p)(\mathbf{m}^{(s)}+\mathbf{m}^{(s')})\mathbf{R}}\,|A^{(s)}A^{(s')}|^2\left(\mathbf{m}^{(s)}\mathbf{m}^{(s')}\right)^2.$$

When all non-coherent elementary surface waves propagate in the same direction ($\mathbf{m}^{(s)}$=$\mathbf{m}$), the spectral light beam tensor

$$\Phi_3^{pl} = \frac{e^{-2\,\mathrm{Im}(k_p)\mathbf{m}\mathbf{R} - 2\kappa_d z}}{|\varepsilon_d|^2}\left(\sum_s |A^{(s)}|^2\right)\mathbf{b} \otimes \mathbf{b}^* \qquad (12)$$

is proportional to the dyad $\mathbf{b} \otimes \mathbf{b}^*$, which indicates that the plasmonic beam is completely polarized. This means that the partially polarized beam excites fully polarized SPPs.

At the circular interface of a fiber, localized waves can emerge as the waveguide modes characterized by the propagation constant β and azimuthal number ν. These quantities are the analog of the wavevector for planar geometry. Remembering that the partially polarized plasmons at the metal-dielectric interface appear only owing to the distinct directions of the wavevectors, we can suppose that the circular plasmons arise due to the random azimuthal numbers ν. Thus the electric field of the s-th elementary wave could be presented in the form

$$\mathbf{E}^{(s)} = e^{i\nu^{(s)}\varphi} A^{(s)} \mathbf{e}^{(s)}(\nu^{(s)}), \qquad (13)$$

where $\mathbf{e}^{(s)}(\nu^{(s)})$ is the polarization vector of the excited mode $\nu^{(s)}$. The polarization vector can be symbolically written in terms of the cylindrical impedance tensors [17] of the core and cladding, Γ_{co} and Γ_{cl}, as $\mathbf{e}^{(s)} = \Gamma_{cl}^{(s)}\left(\Gamma_{cl}^{(s)} - \Gamma_{co}^{(s)}\right)\mathbf{p} \equiv \hat{\chi}^{(s)}\mathbf{p}$, where $\mathbf{p}$ is an arbitrary vector $(\left(\Gamma_{cl}^{(s)} - \Gamma_{co}^{(s)}\right)\mathbf{p} \neq 0)$. The two-dimensional plasmonic beam tensor is constructed of the electric fields projected onto the fiber interface:

$$\Phi_2 = \sum_s \mathbf{I}_r\mathbf{E}^{(s)} \otimes \mathbf{I}_r\mathbf{E}^{(s)*} = \sum_s |A^{(s)}|^2\,\hat{\chi}^{(s)}\mathbf{p} \otimes \mathbf{p}\hat{\chi}^{(s)+},$$

where $\mathbf{I}_r = \mathbf{1} - \mathbf{e}_r \otimes \mathbf{e}_r$. When all surface waves have the same azimuthal number $\nu^{(s)}$= ν, the introduced tensor $\hat{\chi}^{(s)}$ does not depend on s and, hence, $\Phi_2 \propto \hat{\chi}\mathbf{p} \otimes \mathbf{p}\hat{\chi}^+$ corresponds to the completely polarized light.

In conclusion, we have inspected the spectral plasmonic beam tensor (coherence matrix) treating the SPP field as consequently three-dimensional and two-dimensional fields. In our opinion, the two-dimensional beam tensor for light localized at the metal-dielectric interface is beneficial, because the light beam tensor constructed of magnetic fields of non-coherent SPPs is basically two-dimensional.

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References

[1] G.A. Siviloglou J. Broky, A. Dogariu, D.N. Christodoulides, Phys. Rev. Lett. **99**, 213901 (2007).

[2] G.A. Siviloglou, D. N. Christodoulides, Opt. Lett. **32**, 979 (2007).

[3] J. Durnin, JOSA A **4**, 651 (1987).

[4] A. April, JOSA A **28**, 2100 (2011).

[5] E. Wolf, Nuovo Cimento **12**, 884 (1954a).

[6] T. Setala, A. Shevchenko, M. Kaivola, A.T. Friberg, Phys. Rev. E **66**, 016615 (2002).

[7] J. Ellis, A. Dogariu, S. Ponomarenko, E.Wolf, Opt. Commun. **248**, 333 (2005).

[8] C. Brosseau, A. Dogariu, Progress in Optics **49**, 315 (2006).

[9] A. Norrman, T. Setala, A.T. Friberg, Proc. of SPIE, **8011**, 80113O (2011).

[10] R. Carminati, J.-J. Greffet, Phys. Rev. Lett., 82, 1660 (1999).

[11] C.J. Zapata-Rodrigues, S. Vukovic, M.R. Belic, D. Pastor, J.J. Miret, Opt. Express, **19**, 19572 (2011).

[12] A. Minovich, A.E. Klein, N. Janunts, T. Pertsch, D.N. Neshev, Y.S. Kivshar, Phys. Rev. Lett. **107**, 116802 (2011).

[13] F.I. Fedorov, Zh. Prikl. Spectrosk. **2**, 523 (1965).

[14] F.I. Fedorov, *Theory of Gyrotropy* (Minsk, Nauka i Technika 1976).

[15] A.V. Lavrinenko, I.I. Gancheryonok, T. Dreier, J. Opt. B: Quantum Semiclass. Opt. **3**, S202 (2001).

[16] S.A. Maier, *Plasmonics: Fundamentals and Applications* (Berlin, Springer, 2007).

[17] A.V. Novitsky and L.M. Barkovsky, J. Phys. A: Math. Gen. **38**, 391 (2005).

