Influence of a double Fano structure on pulse propagation in an autoionizing medium

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Abstract—We discuss the propagation of a short laser pulse in an autoionizing medium with a double Fano structure. By solving the coupled equations for atoms and fields simultaneously, we can consider the influence of a double Fano structure on a pulse laser in the transmission process.

The interaction of radiation with a quantum system containing a continuum, coupled by an optical field to some bound state, typically leads to unrecoverable loss of bound-state population. However, by using coherent radiation the bound state – continuum interaction can be effectively controlled. The existence of quantum interference effects in atom-field couplings can lead to such interesting phenomena involving bound states and structured continuum as electromagnetically induced transparency[1-3], laser-induced continuum structure generation[4-5] population transfer through a structured continuum[6] or lasing without population inversion[7-8]. It should be mentioned that the optical properties of AI systems have also been studied in a context of lasing without inversion problems[7-8].

The propagation of a short laser pulse in an autoionizing medium with a Fano profile has been considered in[9]. In the systems containing bound states, the introduction of dissipative processes usually destroys several interesting coherence phenomena. In this paper, we discuss the propagation of a short laser pulse in an autoionizing medium with a double Fano structure. By solving the coupled equations for atoms and fields simultaneously, we can consider the influence of a double Fano structure on a pulse laser in the transmission process.

The autoionization system that we consider is shown in Fig.1 and it is a generalization of the model discussed by Paspalakis, Kylstra, Knight proposed in [9]. Contrary to the model considered in [9] and including a single autoionizing level, our model involves two AI levels |1⟩, |2⟩ with energies $E_1$ and $E_2$, respectively. Moreover, these states are embedded in a flat continuum. AI levels interact with the continuum |E⟩ by configurational interaction and this interaction is described by the parameters $V_1$ and $V_2$, respectively. As it has been emphasized in [9], this configurational coupling is responsible for the interference which leads to a transparency phenomenon. The AI states and the continuum are coupled with the state |0⟩ by an external laser field.

We assume that the laser field is described classically by a time- and space-dependent electric field can be written as:

$$E(z,t) = \mathbb{E}(z,t) = \mathbb{E} \left[ F_\theta f(z,t) \exp[i(\omega t - kz) + c.c.] \right],$$

where $\omega$ is the angular frequency, $k$ is the wave number. Moreover, $\mathbb{E}$ is the polarization vector, $F_\theta$ is the electric field amplitude and finally $f(z,t)$ is the dimensionless pulse envelope of the laser field.

The Hamiltonian of our system can be written as:

$$\hat{H} = \sum_{m=1,2} E_m |m\rangle \langle m| + \int_0^\infty d\omega |\mathbb{E}(z,\omega)\rangle \langle \mathbb{E}(z,\omega)| + \int_0^\infty d
$$

Furthermore, $E_m$ is the unperturbed energy of level |m⟩, $\mathbb{E}$ is the energy of the continuum states |$c$⟩, $V_{1c} = \langle 1|V_1|c,\omega\rangle$, $V_{2c} = \langle 2|V_2|c,\omega\rangle$, $\mu_{1c} = \langle 0|\mathbb{E}|c,\omega\rangle$ and $\mu_{2c} = \langle 0|\mathbb{E}|c,\omega\rangle$ are the dipole matrix elements, with $\mu$ being the dipole moment operator.
To describe the time-evolution of our system we use the following wave function. It can be expressed in the form:

$$\psi(z,t) = \sum_{n=0,2} a_n(z,t)e^{-iE_nt}|n\rangle + \int_0^\infty d\alpha(z,t)e^{-iE_{\alpha}t}|\alpha\rangle. \quad (3)$$

We substitute this expression into the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}\psi(z,t) = \hat{H}\psi(z,t) \quad (4)$$

to obtain the following set of equations:

$$i\dot{a}_0 = -\mu_0E(z,t)a_0(z,t)e^{-(E_0-E_0)t} - \mu_0E(z,t)a_2(z,t)e^{-(E_2-E_0)t} - \int_0^\infty d\mu\mu_0E(z,t)a_\mu(z,t)e^{-(E_\mu-E_0)t}, \quad (5)$$

$$i\dot{a}_1 = -\mu_0E(z,t)a_0(z,t)e^{-(E_0-E_0)t} + \int_0^\infty d\mu\mu_0E(z,t)a_\mu(z,t)e^{-(E_\mu-E_0)t}, \quad (6)$$

$$i\dot{a}_2 = -\mu_2E(z,t)a_0(z,t)e^{-(E_0-E_0)t} + \int_0^\infty d\mu\mu_2E(z,t)a_\mu(z,t)e^{-(E_\mu-E_0)t} - \mu_0E(z,t)a_0(z,t)e^{-(E_0-E_0)t}. \quad (7)$$

We eliminate further the continuum amplitude by solving formally Eq. (8), then we have:

$$a_\alpha(z,t) = \int_0^\infty d\alpha a_\alpha(z,t)e^{-iE_\alpha t} + \int_0^\infty d\alpha' a_\alpha(z,t)e^{-iE_\alpha' t}.$$  

Substituting the Eq. (9) into Eqs. (5), (6) and (7) we obtain the following equations (after making the rotating wave and the Markov approximation):

$$i\frac{\partial}{\partial t}C(z,t) = \hat{H}(z,t)C(z,t),$$

where

$$\hat{H}(z,t) = \begin{pmatrix}
\frac{\delta_{E_0-E_0}}{2}f(\xi(z,t)) & -\frac{1}{2}(E_0+\xi(z,t))S_0(z,t) & -\frac{1}{2}(E_0+\xi(z,t))S_0(z,t) \\
-\frac{1}{2}(E_0+\xi(z,t))S_0(z,t) & \frac{\delta_{E_0-E_2}}{2}f(\xi(z,t)) & -\frac{1}{2}(E_0+\xi(z,t))S_0(z,t) \\
-\frac{1}{2}(E_0+\xi(z,t))S_0(z,t) & -\frac{1}{2}(E_0+\xi(z,t))S_0(z,t) & \frac{\delta_{E_0-E_2}}{2}f(\xi(z,t))
\end{pmatrix}.$$  

$$C(z,t) = \begin{pmatrix} c_1(z,t), c_1(z,t), c_2(z,t) \end{pmatrix}.$$  

The width $\Gamma_0 = 2\pi\mu_0E_0 \gamma_0$ denotes the ionization rate of $|0\rangle$ and $\delta E_\alpha = -\frac{P}{\alpha - \delta_0}$ is the AC Stark shift of level $|0\rangle$ (the symbol P denotes the Cauchy principal part). The laser detunings are $\Delta_1 = E_1 - \delta E_1 - E_0 + \delta$ and $\Delta_2 = E_2 - E_0 - \omega$, whereas $q_1$ and $q_2$ are Fano asymmetry parameters [3,10]. It should be noted that the autoionization rates $\Gamma_1, \Gamma_2$ and shifts $\delta E_1, \delta E_2$ appear as a result of coupling the states $|1\rangle$ and $|2\rangle$ to the continuum. The non-Hermitian Hamiltonian given above is more complex than that considered in [6]. Due to its complicated form we shall concentrate our attention on numerical solutions only.

In order to study the propagation of short laser pulses in the medium, the Maxwell wave equation is required, which in the slowly-varying-envelope approximation has the form

$$\left[\frac{\partial}{\partial \xi}f(\xi,t) + \frac{1}{c} \frac{\partial}{\partial t}f(\xi,t)\right]e^{i(\omega t-\xi)} = \frac{-2\mu_0}{c}P(z,t). \quad (10)$$

The macroscopic polarization of the medium is given by, assuming only homogeneous broadening,

$$P(z,t) = N\left[\mu_0\rho_0(z,t)a_0(z,t)e^{-(E_0-E_0)t} + \mu_0\rho_0(z,t)x\right]a_0(z,t)e^{-(E_0-E_0)t}, \quad (11)$$

with $N$ being the atomic density. Substituting Eq. (9) into Eq. (11) and after making the Markov approximation, we obtain the following equation for the laser field envelope:

$$\frac{\partial}{\partial \xi}f(\xi,t) + \frac{1}{c} \frac{\partial}{\partial t}f(\xi,t) = ia\left[\delta E_0 + \frac{i}{2}\Gamma_0\right]f(\xi,t)f(\xi,t)^* + \frac{1}{2c^2}\left(1 - q_1\right)c_1\xi(z,t)c_1(z,t)$$  

$$\times \left(1 - q_1\right)c_1\xi(z,t)c_1(z,t) + \frac{1}{2c^2}\left(1 - q_1\right)c_1\xi(z,t)c_1(z,t). \quad (12)$$

Here $\alpha = 2\pi N\omega_0 / c E_0^2$ is proportional to the propagation constant. We transform the resulting equations into the local retarded frame where $\tau = t - z/c$ and $\xi = z$, and obtain the coupled Maxwell-Schrödinger equations that govern the evolution of the system in the following form:

$$i\frac{\partial}{\partial \tau}C(\xi,\tau) = \hat{H}(\xi,\tau)C(\xi,\tau), \quad (13)$$

$$\frac{\partial}{\partial \tau}f(\xi,\tau) = ia\left[\delta E_0 + \frac{i}{2}\Gamma_0\right]f(\xi,\tau)^*f(\xi,\tau) + \frac{1}{c}\left(1 - q_1\right)c_1\xi(z,\tau)c_1(z,\tau)$$  

$$\times \left(1 - q_1\right)c_1\xi(z,\tau)c_1(z,\tau). \quad (14)$$

In general, the coupled equations given by Eqs. (13) and (14) cannot be solved analytically, numerical computations are necessary. The temporal profiles of the field at the entrance to the medium is assumed to be

$$f(\xi = 0,\tau) = \sin^2\left(\frac{\pi \tau}{\tau_p}\right),$$

with $\tau_p$ being the pulse duration.

We demonstrate the degenerate case ($E_1 = E_2$) in Fig. 2 with additional assumptions $q_1 = q_2$ and $\Gamma_1 = \Gamma_2$. From this figure we see that the pulse propagates without loss, but with a change of group velocity. So we obtain the same results as in [9], however instead of the parameters $q, \Gamma$ given in [9] we have $q_1 + q_2$ and $\Gamma_1 + \Gamma_2$, respectively. In this case two AI states play the same role and they are indistinguishable.
We consider now the non-degenerate case when two AI states are not identical and their positions are determined by detunings $\Delta_1 = -4$ and $\Delta_2 = -2$. Figures 3a and 3b present the maximal value of the function $|\tilde{f}(\xi, t)|^2$ at the end of the propagation process as a function of the Fano parameters. In Figure 3a we change the parameter $q_2$ and set $q_1 = 4$, and vice versa for Fig. 3b. From these figure we see that pulse attenuation caused by absorption appear during pulse propagation. However, sometimes we can eliminate the influence of absorption. We see that for $q_1 = 7.3$ (Fig. 3a) and $q_1 = 1$ (Fig. 3b) attenuation can be omitted. Therefore we can always choose a pair of values for the parameters $q_1$ and $q_2$ for which the absorption is neglected.

In this paper, we considered the propagation of a short laser pulse in an autoionizing medium. The main result of this paper concerns the proposed model involving two AI states. This model is an extension of that considered by Paspalakis et al. [9]. We showed that by a proper choice of Fano parameters involved in the problem one can almost completely eliminate absorption in pulse propagation. It means that transparency can appear in the medium. As the population trapping phenomena are strongly related to transparency effects, one can conclude that by the proper choice of the parameters describing the system the population trapping (or the existence of the dark states) can occur in the atomic system.

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References