Influence of a double Fano structure on pulse propagation in an autoionizing medium

Thanh Thuy Do, Dinh Xuan Khoa, and Bui Dinh Thuan*

Faculty of Physics and Technology, Vinh University, Vietnam

Received August 27, 2016; accepted September 26, 2016; published September 30, 2016

Abstract—We discuss the propagation of a short laser pulse in an autoionizing medium with a double Fano structure. By solving the coupled equations for atoms and fields simultaneously, we can consider the influence of a double Fano structure on a pulse laser in the transmission process.

The interaction of radiation with a quantum system containing a continuum, coupled by an optical field to some bound state, typically leads to unrecoverable loss of bound-state population. However, by using coherent radiation the bound state – continuum interaction can be effectively controlled. The existence of quantum interference effects in atom-field couplings can lead to such interesting phenomena involving bound states and structured continum as electromagnetically induced transparency [1-3], laser-induced continuum structure generation [4-5] population transfer through a structured continumm [6] or lasing without population inversion [7-8]. It should be mentioned that the optical properties of AI systems have also been studied in a context of lasing without inversion problems [7-8].

The propagation of a short laser pulse in an autoionizing medium with a Fano profile has been considered in [9]. In the systems containing bound states, the introduction of dissipative processes usually destroys several interesting coherence phenomena. In this paper, we discuss the propagation of a short laser pulse in an autoionizing medium with a double Fano structure. By solving the coupled equations for atoms and fields simultaneously, we can consider the influence of a double Fano structure on a pulse laser in the transmission process.

The autoionization system that we consider is shown in Fig.1 and it is a generalization of the model discussed by Paspalakis, Kylstra, Knight proposed in [9]. Contrary to the model considered in [9] and including a single auto-ionizing level, our model involves two AI levels $|1\rangle$, $|2\rangle$ with energies E_1 and E_2, respectively. Moreover, these states are embedded in a flat continuum. AI levels interact with the continuum $|E\rangle$ by configurational interaction and this interaction is described by the parameters V_1 and V_2, respectively. As it has been emphasized in [9], this configurational coupling is responsible for the interference which leads to a transparency phenomenon. The AI states and the continuum are coupled with the state $|0\rangle$ by an external laser field.

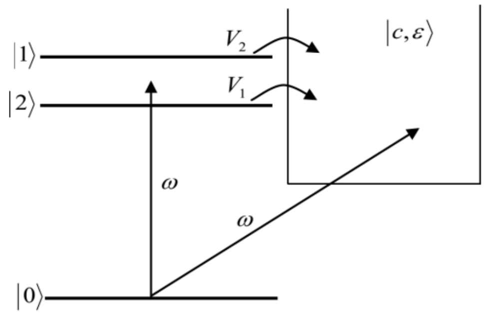

Fig. 1. The level and coupling scheme.

We assume that the laser field is described classically by a time - and space-dependent electric field can be written as:

$$\vec{E}(z,t)=\vec{\varepsilon}E(z,t)=\vec{\varepsilon}\left[F_0 f(z,t)\exp\left[i(\omega t-kz)+c.c.\right]\right], \quad (1)$$

where ω is the angular frequency, k is the wave number. Moreover, $\vec{\varepsilon}$ is the polarization vector, F_0 is the electric field amplitude and finally $f(z,t)$ is the dimensionless pulse envelope of the laser field.

The Hamiltonian of our system can be written as:

$$\hat{H}=\sum_{m=0,1,2}E_m|m\rangle\langle m|+\int_0^\infty d\varepsilon\varepsilon|c,\varepsilon\rangle\langle c,\varepsilon|+\left[\int_0^\infty d\varepsilon V_{1\varepsilon}|1\rangle\langle c,\varepsilon| + \int_0^\infty d\varepsilon V_{2\varepsilon}|2\rangle\langle c,\varepsilon| -\mu_{01}E(z,t)|0\rangle\langle 1|-\mu_{02}E(z,t)|0\rangle\langle 2| - \int_0^\infty d\varepsilon\mu_{0\varepsilon}|0\rangle\langle c,\varepsilon|+H.c.\right], \quad (2)$$

where E_m is the unperturbed energy of level $|m\rangle$,
ε is the energy of the continuum states $|\varepsilon\rangle$,
$V_{1\varepsilon}=\langle 1|V_1|c,\varepsilon\rangle$; $V_{2\varepsilon}=\langle 2|V_2|c,\varepsilon\rangle$,
$\mu_{0m}=\langle 0|\vec{\mu}.\vec{\varepsilon}|m\rangle$ and $\mu_{0\varepsilon}=\langle 0|\vec{\mu}.\vec{\varepsilon}|c,\varepsilon\rangle$ are the dipole matrix elements, with $\vec{\mu}$ being the dipole moment operator.

* E-mail: thuanbd@vinhuni.edu.vn

To describe the time-evolution of our system we use the following wave-function. It can be expressed in the form:

$$\psi(z,t)=\sum_{m=0,1,2} a_m(z,t)e^{-iE_m t}|m\rangle+\int_0^\infty d\varepsilon a_\varepsilon(z,t)e^{-i\varepsilon t}|c,\varepsilon\rangle. \tag{3}$$

We substitute this expression into the time – dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi(z,t)\rangle=\hat{H}|\psi(z,t)\rangle \tag{4}$$

to obtain the following set of equations:

$$i\dot{a}_0=-\mu_{01}E(z,t)a_1(z,t)e^{-i(E_1-E_0)t}-\mu_{02}E(z,t)a_2(z,t)e^{-i(E_2-E_0)t} -\int_0^\infty d\varepsilon\mu_{0\varepsilon}E(z,t)a_\varepsilon(z,t)e^{-i(\varepsilon-E_0)t}, \tag{5}$$

$$i\dot{a}_1=-\mu_{10}E^*(z,t)a_0e^{i(E_1-E_0)t}+\int_0^\infty d\varepsilon V_{1\varepsilon}a_\varepsilon e^{-i(\varepsilon-E_1)t}, \tag{6}$$

$$i\dot{a}_2=-\mu_{20}E^*(z,t)a_0e^{i(E_2-E_0)t}+\int_0^\infty d\varepsilon V_{2\varepsilon}a_\varepsilon e^{-i(\varepsilon-E_2)t}, \tag{7}$$

$$i\dot{a}_\varepsilon=V_{\varepsilon1}a_1e^{i(\varepsilon-E_1)t}+V_{\varepsilon2}a_2e^{i(\varepsilon-E_2)t}-\mu_{\varepsilon0}E^*(z,t)a_0e^{i(\varepsilon-E_0)t}. \tag{8}$$

We eliminate further the continuum amplitude by solving formally Eq. (8), then we have:

$$a_\varepsilon(z,t)=i\int_0^t dt'\mu_{\varepsilon0}F_0^*f^*(z,t')a_0(z,t')e^{i(\varepsilon-E_0-\omega)t'+ikz} -\int_0^t dt'V_{\varepsilon1}a_1(z,t')e^{i(\varepsilon-E_1)t'} -\int_0^t dt'V_{\varepsilon2}a_2(z,t')e^{i(\varepsilon-E_2)t'}. \tag{9}$$

Substituting the Eq. (9) into Eqs. (5), (6) and (7) we obtain the following equations (after making the rotating wave and the Markov approximation):

$$i\frac{\partial}{\partial t}C(z,t)=H(z,t)C(z,t),$$

where

$$H(z,t)=\begin{pmatrix} \left(\delta E_0-\frac{i}{2}\Gamma_0\right)|f(z,t)|^2 & -\frac{1}{2}(q_1+i)\sqrt{\Gamma_0\Gamma_1}f(z,t) & -\frac{1}{2}(q_2+i)\sqrt{\Gamma_0\Gamma_1}f(z,t) \\ -\frac{1}{2}(q_1+i)\sqrt{\Gamma_0\Gamma_1}f^*(z,t) & \Delta_1-\frac{i}{2}\Gamma_1 & \delta E_{12}-\frac{i}{2}\sqrt{\Gamma_1\Gamma_2} \\ -\frac{1}{2}(q_2+i)\sqrt{\Gamma_0\Gamma_2}f^*(z,t) & \delta E_{12}-\frac{i}{2}\sqrt{\Gamma_1\Gamma_2} & \Delta_2-\frac{i}{2}\Gamma_2 \end{pmatrix},$$

$C(z,t)=[c_0(z,t), c_1(z,t), c_2(z,t)]^T$.

The width $\Gamma_0=2\pi|\mu_{0\varepsilon}F_0|^2$ denotes the ionization rate of $|0\rangle$ and $\delta E_0=-\mathrm{P}\int_0^\infty d\varepsilon\frac{|V_{o\varepsilon}|^2}{\varepsilon-\omega_0-\omega}$ is the AC Stark shift of level $|0\rangle$

(the symbol P denotes the Cauchy principal part). The laser detunings are $\Delta_1 = E_1 + \delta E_1 - E_0 - \omega$ and $\Delta_2 = E_2 + \delta E_2 - E_0 - \omega$, whereas q_1 and q_2 are Fano asymmetry parameters [3,10]. It should be noted again that the autoionization rates Γ_1, Γ_2 and shifts δE_1, δE_2 appear as a result of coupling the states $|1\rangle$ and $|2\rangle$ to the continuum. The non-Hermitian Hamiltonian given above is more complex than that considered in [6]. Due to its complicated form we shall concentrate our attention on numerical solutions only.

In order to study the propagation of short laser pulses in the medium, the Maxwell wave equation is required, which in the slowly-varying-envelope approximation has the form

$$\left[\frac{\partial}{\partial z}f(z,t)+\frac{1}{c}\frac{\partial}{\partial t}f(z,t)\right]F_0e^{i(\omega t-kz)}=-\frac{2i\pi\omega}{c}P(z,t), \tag{10}$$

The macroscopic polarization of the medium is given by, assuming only homogeneous broadening,

$$P(z,t)=N\Big[\ \mu_{01}a_0(z,t)a_1^*(z,t)e^{i(E_1-E_0)t}+\mu_{02}a_0(z,t)\times \times a_2^*(z,t)e^{i(E_2-E_0)t}\ +\int_0^\infty d\varepsilon\mu_{\varepsilon0}a_0(z,t)a_\varepsilon^*(z,t)e^{i(\varepsilon-E_0)t}\Big], \tag{11}$$

with N being the atomic density. Substituting Eq. (9) into Eq. (11) and after making the Markov approximation, we obtain the following equation for the laser field envelope:

$$\frac{\partial}{\partial z}f(z,t)+\frac{1}{c}\frac{\partial}{\partial t}f(z,t)=i\alpha\Big[\left(\delta E_0+\frac{i}{2}\Gamma_0\right)f(z,t)|c_0(z,t)|^2 +\frac{1}{2}\sqrt{\Gamma_0\Gamma_1}(i-q_1)c_0(z,t)c_1^*(z,t) +\frac{1}{2}\sqrt{\Gamma_0\Gamma_2}(i-q_2)c_0(z,t)c_2^*(z,t)\Big], \tag{12}$$

here $\alpha=2\pi N\omega/cF_0^2$ is proportional to the propagation constant. We transform the resulting equations into the local retarded frame where $\tau = t - z/c$ and $\xi = z$, and obtain the coupled Maxwell-Schrödinger equations that govern the evolution of the system in the following form:

$$i\frac{\partial}{\partial\tau}C(\xi,\tau)=H(\xi,\tau)C(\xi,\tau), \tag{13}$$

$$\frac{\partial}{\partial\xi}f(\xi,\tau)=i\alpha\Big[\left(\delta E_0+\frac{i}{2}\Gamma_0\right)f(\xi,\tau)|c_0(\xi,\tau)|^2+\frac{1}{2}\sqrt{\Gamma_0\Gamma_1}\times \times(i-q_1)c_0(\xi,\tau)c_1^*(\xi,\tau)++\frac{1}{2}\sqrt{\Gamma_0\Gamma_2}(i-q_2)c_0(\xi,\tau)c_2^*(\xi,\tau)\Big]. \tag{14}$$

In general, the coupled equations given by Eqs. (13) and (14) cannot be solved analytically, numerical computations are necessary. The temporal profiles of the field at the entrance to the medium is assumed to be

$$f(\xi=0,\tau)=\sin^2\left(\frac{\pi\tau}{\tau_p}\right),$$

with τ_p being the pulse duration.

We demonstrate the degenerate case ($E_1 = E_2$) in Fig. 2 with additional assumptions $q_1 = q_2$ and $\Gamma_1=\Gamma_2$. From this figure we see that the pulse propagates without loss, but with a change of group velocity. So we obtain the same results as in [9], however instead of the parameters q, Γ_1 given in [9] we have q_1+q_2 and $\Gamma_1+\Gamma_2$, respectively. In this case two AI states play the same role and they are indistinguishable.

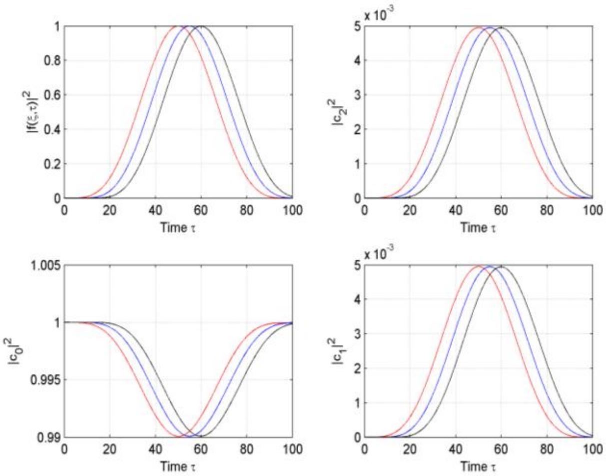

Fig. 2. Magnitude squared of the pulse envelope and the atomic populations as a function of τ for different values of ξ, with $\alpha\xi=0$ (red curves), $\alpha\xi=\alpha\xi_{max}/2$ (blue curves), and $\alpha\xi=\alpha\xi_{max}=1000$ (black curves) .The parameters used are $\delta E_{21} = 0$, $\tau = 100$, $\Gamma_0= 2x10^{-2}$, $\delta E_0= -5.10^{-2}$, $q_1 = q_2 =5$ and $\Delta_1 = \Delta_2 = -5$. The parameters are given in units of Γ_1.

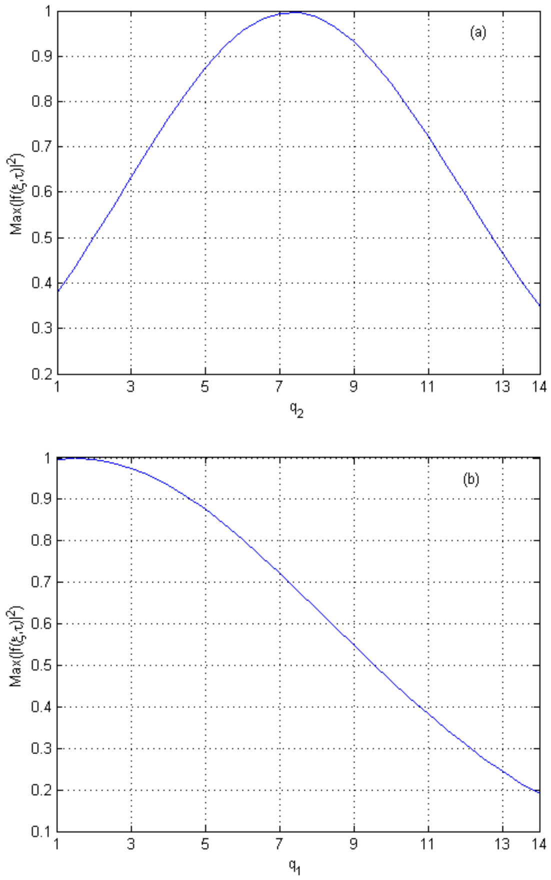

Fig. 3. The maximal value of the function $|f(\xi,\tau)|^2$ at the end of the propagation process as a function of the Fano parameters. The parameters used are $\delta E_{21} = 0$, $\tau = 100$, $\Gamma_0= 5.10^{-3}$, $\delta E_0= -5.10^{-3}$, $\Gamma_2 =\Gamma_1$, $\Delta_1 =-4$, $\Delta_2 = -2$. (a) $q_1 = 4$ and q_2 takes the values from 1 to 14; (b) $q_2 = 4$ and q_1 takes the values from 1 to 14.

We consider now the non-degenerate case when two AI states are not identical and their positions are determined by detunings $\Delta_1 = -4$ and $\Delta_2 = -2$. Figures 3a and 3b present the maximal value of the function $|f(\xi,\tau)|^2$ at the end of the propagation process as a function of the Fano parameters. In Figure 3a we change the parameter q_2 and set $q_1 = 4$, and vice versa for Fig. 3b. From these figure we see that pulse attenuation caused by absorption appear during pulse propagation. However, sometimes we can eliminate the influence of absorption. We see that for $q_2 = 7.3$ (Fig. 3a) and $q_1 = 1$ (Fig. 3b) attenuation can be omitted. Therefore we can always choose a pair of values for the parameters q_1 and q_2 for which the absorption is neglected.

In this paper, we considered the propagation of a short laser pulse in an autoionizing medium. The main result of this paper concerns the proposed model involving two AI states. This model is an extension of that considered by Paspalakis *et al.* [9]. We showed that by a proper choice of Fano parameters involved in the problem one can almost completely eliminate absorption in pulse propagation. It means that transparency can appear in the medium. As the population trapping phenomena are strongly related to transparency effects, one can conclude that by the proper choice of the parameters describing the system the population trapping (or the existence of the dark states) can occur in the atomic system.

This research was funded by the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under the grant number 103.03-2014.62.

References

[1] A. Raczyński, M. Rzepecka, J. Zaremba, S. Zielińska-Kaniasty, Opt. Commun. **266**, 552 (2006).
[2] T. Bui Dinh, W. Leoński , V. Cao Long, J. Perina Jr, Eur. Phys. J. D, **68**, 150 (2014).
[3] T. Bui Dinh, W. Leoński, V. Cao Long, J. Perina Jr, Opt. Appl. **43,** 471 (2013).
[4] C. Long Van, Structured continuum in various optical phenomena, in *Physics and applications I: Quantum Optics* (Ed. Wiesław Leoński, University of Zielona Góra Press 2012).
[5] M. Shapiro, Phys.Rev.A **75**, 013424 (2007).
[6] E. Paspalakis, P.L. Knight, J. Phys. B **31**, 2753 (1998).
[7] S.Y. Zhu, E.E. Fill, Phys. Rev. A **42**, 5684 (1990).
[8] S.E. Harris, Phys. Rev. A **62**, 1033 (1989).
[9] E. Paspalakis, N.J. Kylstra, P.L. Knight, Phys. Rev. A **60,** 642 (1999).
[10] W. Leoński, R. Tanaś, S. Kielich, J. Opt. Am. B **4**, 72 (1987).

