# Tunable sinusoidal Phase Gratings and sinusoidal Phase Zone Plates

Jorge Ojeda-Castañeda,*[1] Cristina M. Gómez-Sarabia, [2] Miguel Torres-Cisneros, [1] Luis M. Ledesma-Carrillo, [1] Rafael Guzmán-Cabrera, [3] and Cipriano Guzmán-Cano [1]

[1]*Electronics Department, DICIS, Universidad de Guanajuato, Salamanca, Guanajuato, 36850, México*
[2]*Digital Arts Department, DICIS, Universidad de Guanajuato, Salamanca, Guanajuato, 36850, México*
[3]*Electrical Engineering Department, DICIS, Universidad de Guanajuato, Salamanca, Guanajuato, 36850, México*

Received April 17, 2017; accepted June 12, 2017; published June 30, 2017

**Abstract**—We disclose the use of a suitable pair of phase-only masks for generating sinusoidal phase gratings, or sinusoidal phase zone plates, with tunable optical path delays. We show that these gratings can have either a 1-D or a 2-D radial structure.

There are currently several competing techniques for implementing lenses with tunable optical power [1-5]. These developments can be related to the efforts addressed to control fully (not only for extending) field depth; without modifying the size pupil aperture at [6, 7].

Here, we unveil the use of a pair of suitable phase-only masks for generating 1-D rectangular, phase gratings, or 2-D circular phase gratings and 2-D circular, phase zone plates. We show that, by a suitable mechanical alteration, these optical elements can have tunable optical path delays.

Our current proposal extends the concept that was first proposed by Lohmann [8-11] and by Alvarez [12-14].

To our end, first, we discuss our proposal for generating 1-D phase gratings. Next, we present our proposal for producing radial phase gratings and radial, phase zone plates. And finally, we summarize our contribution.

In Figure 1 we depict the optical setup for illuminating two phase masks with a collimated beam.

The complex amplitude transmittance of the first mask is

$$g_1(x) = \exp\left[i\,2\pi\,a\sin\left(2\pi\frac{x}{d}\right)\right]\mathrm{rect}\left(\frac{x}{L}\right). \quad (1)$$

In Eq. (1) we represent a sinusoidal phase grating with a fixed optical path difference *a* and a fixed period *d*. The last term of Eq. (1) represents the finite width, L, of the mask.

* E-mail: jojedacas@ugto.mx

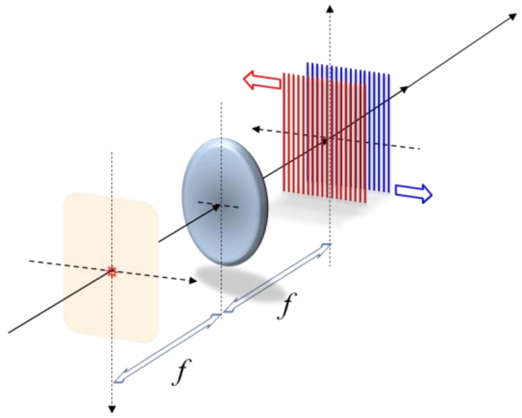


Fig. 1. Schematics of the optical setup for generating a 1-D sinusoidal phase grating, with a tunable optical path

For the second mask, the complex amplitude transmittance is

$$g_2(x) = g_1{}^*(x)\,. \quad (2)$$

Next, the two above masks are placed in close contact for setting a pair. By introducing a controllable lateral displacement *D*, between the members of the pair, the overall complex amplitude transmittance is

$$g(x;D) = g_1\left(x+\frac{D}{2}\right)g_1{}^*\left(x-\frac{D}{2}\right)rect\left(\frac{x}{X}\right). \quad (3)$$

In Eq. (3) we consider that the optical system has an overall window, its width is equal to *X*; where *X* is smaller than L; say $X \leq L - D$. Consequently, the maximum value for the lateral displacement *D* should be less than L / 2.

By substituting Eq. (1) in Eq. (3), the overall complex amplitude transmittance reads

$$g(x;D) = \exp\left\{i\,2\pi\left[2\,a\sin\left(\pi\frac{D}{d}\right)\right]\cos\left(2\pi\frac{x}{d}\right)\right\}rect\left(\frac{x}{X}\right). \quad (4)$$

 

It is apparent from Eq. (4) that the optical path delay varies as a sinusoidal function of the lateral displacement *D*. We recognize also that the period d remains constant.

Equivalently, by using the Jacobi-Bessel expansion, we can express the overall complex amplitude transmittance as

$$g(x;D)=\sum_{n=-\infty}^{\infty}(i)^n J_n\left[2a\sin\left(\pi\frac{D}{d}\right)\right]\exp\left(i2\pi\frac{n}{d}x\right)\ rect\left(\frac{x}{X}\right). \qquad (5)$$

It is clear from Eq. (5) that by introducing a controllable lateral displacements *D*, one can shape the Fourier coefficients of the grating.

Now, we focus our attention on the generation of periodical phase elements with circular symmetry; as depicted in Fig. 2. For this application, we follow the proposals outlined in references [15] and [16-18].

Now, for the first element of the pair, the complex amplitude transmittance is

$$g_1(r,\theta)=\exp\left[i2\pi a\left(\frac{\theta}{2\pi}\right)\sin\left(2\pi\left(\frac{r}{p}\right)\right)\right]circ\left(\frac{r}{R}\right). \qquad (6)$$

In Eq. (6), we denote as ($r$, $\theta$) the polar coordinates at the plane containing the mask. Now, the letter *p* denotes the radial period of the circular grating. We represent the finite support of the mask by using the circ function, which is equal to unity only inside a circle of radius *R*, otherwise the circ function is equal to zero.

For the second element of the pair, the complex amplitude transmittance i

$$g_2(r,\theta)=g_1{}^*(r,\theta). \qquad (7)$$

As before, the two above masks are placed in close contact for setting a pair. However, now we introduce a controllable in-plane rotation $\beta$, between the members of the pair. Then, the overall complex amplitude transmittance is

$$g(r,\theta;\beta)=g_1\left(r,\theta+\frac{\beta}{2}\right)g_1{}^*\left(r,\theta-\frac{\beta}{2}\right)circ\left(\frac{r}{R}\right). \qquad (8)$$

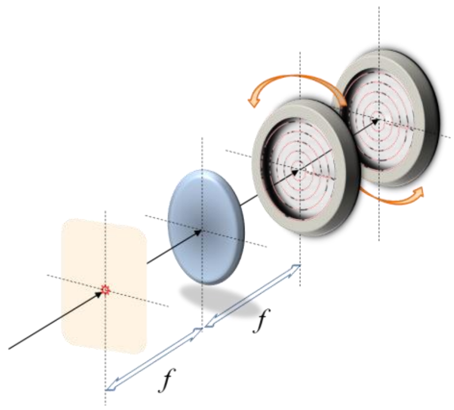


Fig. 2. Optical setup for generating a radial phase grating, with a sinusoidal profile and with tunable optical path delays.

By substituting Eq. (6) in Eq. (8), the overall complex amplitude transmittance reads

$$g(r,\theta;\beta)=\exp\left[i(a\beta)\sin\left(2\pi\left(\frac{r}{p}\right)\right)\right]circ\left(\frac{r}{R}\right). \qquad (9)$$

It is apparent from Eq. (9) that the optical path delay varies as a linear function of the rotation angle $\beta$. We note that the generated grating maintains the same period *p*. For the sake of completeness of our current discussion, we recognize that the Jacobi-Bessel expansion of Eq. (9) reads

$$g(r,\theta;\beta)=\sum_{n=-\infty}^{\infty}J_n[a\beta]\ \exp\left(i2\pi\frac{n}{p}r\right)circ\left(\frac{r}{R}\right). \qquad (10)$$

From Eq. (10) it is clear that by controlling the in-plane rotation angle $\beta$, one can control the Fourier coefficients of the circular grating.

For our final application, we consider the optical setup that is shown in Fig. 3. For the first element of the pair, the complex amplitude transmittance is

$$g_1(r,\theta)=\exp\left[i2\pi a\left(\frac{\theta}{2\pi}\right)\sin\left(2\pi\left(\frac{r}{p}\right)^2\right)\right]circ\left(\frac{r}{R}\right). \qquad (11)$$

 

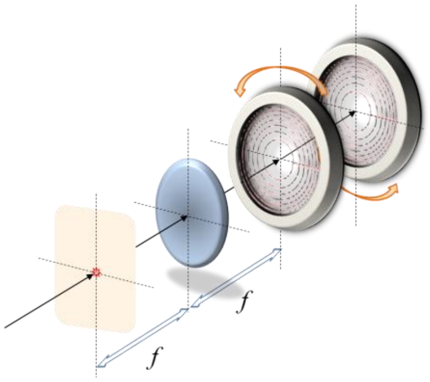


Fig. 3. Schematic diagram of the optical system for generating a radial zone plate, with a sinusoidal phase profile, and a tunable optical path.

For the second element of the pair, the complex amplitude transmittance is the complex conjugate of the expression in Eq. (11). We repeat the procedure outlined before for setting the pair. The overall complex amplitude transmittance is

$$g(r,\theta;\beta) = g_1\left(r,\theta+\frac{\beta}{2}\right) g_1{}^*\left(r,\theta-\frac{\beta}{2}\right) \mathrm{circ}\left(\frac{r}{R}\right). \qquad (12)$$

By substituting Eq. (11) in Eq. (12) we obtain

$$g(r,\theta;\beta) = \exp\left[i(a\beta)\sin\left(2\pi\left(\frac{r}{p}\right)^2\right)\right]\mathrm{circ}\left(\frac{r}{R}\right). \qquad (13)$$

From Eq. (13) we note that the optical path delay varies as a linear function of the rotation angle β. As before, the circular phase variations maintain the same period p. The Jacobi-Bessel expansion of Eq. (13) is

$$g(r,\theta;\beta) = \sum_{n=-\infty}^{\infty} J_n[a\beta]\ \exp\left(i2\pi\left(\frac{r}{p}\right)^2 n\right)\mathrm{circ}\left(\frac{r}{R}\right). \qquad (14)$$

Consequently, by controlling the in-plane rotation angle β, one can control the Fourier coefficients of the phase-only, zone plate.

For applications of Talbot interferometry [19, 20], and before the invention of structured illumination [21], one of us suggested using a phase grating, for illuminating the sample under test. In this manner, one avoids the generation occluding regions on a sample [22]. Additionally, the use of a phase grating shortens the length of the Talbot interferometer.

Hence, here, we recognize that the previously described gratings and the zone plate can be used for illuminating a sample which is under test in a Talbot interferometer. To this end, it is convenient to image the above described phase structures on the sample under test. This task can be done by employing a suitable varifocal lens [23-25].

Summarizing, we have extended the Lohmann - Alvarez technique for generating periodic phase structures, with a controllable optical path difference.

We have shown that the phase structures can be 1-D rectangular phase gratings, 2-D circular transparent gratings and 2-D phase-only zone plates.

We have reported the formula describing the influence of the proposed technique on the Fourier coefficients of periodic structures.

## References


[1] B. Berge, J. Peseux, Eur. Phys. J. **E 3**, 159 (2000).
[2] H.W. Ren, Y.H. Fan, S. Gauza, S.T. Wu, Appl. Phys. Lett. **84**, 4789 (2004).
[3] T. Martinez, D.V. Wick, D.M. Payne, J.T. Baker, S.R. Restaino, Proc. SPIE **5234**, 375 (2004).
[4] D.Y. Zhang, N. Justis, Y.H. Lo, Opt. Commun. **249**, 175 (2005).
[5] B.H.W. Hendriks, S. Kuiper, M.A.J. Van As, C.A. Renders, T.W. Tukker, Opt. Rev. **12**, 255 (2005).
[6] M. Duocastella, C.B. Arnold, Appl. Phys. Lett. **102**, 06113-1 (2013).
[7] J. Ojeda-Castañeda, J. Enrique, A. Landgrave, C.M. Gómez-Sarabia, Appl. Opt. **47**, E99 (2008).
[8] A.W. Lohmann, British Patent 998, 191 (May 29, 1964).
[9] A.W. Lohmann, Republic Française 1, 398, 351 (June 10, 1964).
[10] A.W. Lohmann, Italy Patent 727, 848 (June 19, 1964).
[11] A.W. Lohmann, Appl. Opt. 9, 1669 (1970).
[12] L.W. Alvarez, U.S. patent 3,305,294 (21 February 1967).
[13] L.W. Alvarez, W.E. Humphrey, U.S. patent 3,507,565 (21 April, 1970).
[14] L.W. Alvarez, J. Am. Optometric Assoc. 49, 24 (1978).
[15] S. Bernet and M. Ritsch-Marte, U.S. patent application 20100134869 A1 (June 3, 2010).
[16] J. Ojeda-Castaneda, S. Ledesma, C.M. Gómez-Sarabia, Phot. Lett. Poland **5**, 20 (2013).
[17] J. Ojeda-Castañeda, C.M. Gómez-Sarabia, S. Ledesma, J. Opt. **43**, 85 (2014).
[18] J. Ojeda-Castaneda, A.L. Barragán-Chávez, C.M. Gómez-Sarabia, Phot. Lett. Poland **7**, 8 (2015).
[19] A.W. Lohmann, D.A. Silva, Optics Comm. **2**, 413 (1971).
[20] S. Yokozeki, T. Suzuki, Appl. Opt. **10**, 1575 (1971).
[21] J. Geng, Adv. Opt. Phot. **3**, 128 (2011).
[22] J. Ibarra, J. Ojeda-Castañeda, Opt. Commun. **96**, 294 (1993).
[23] J. Ojeda-Castaneda, C.M. Gómez-Sarabia, S. Ledesma, Asian J. Phys. **23**, 415 (2014).
[24] J. Ojeda-Castaneda, C.M. Gómez-Sarabia, S. Ledesma, Asian J. Phys. **23,** 535 (2014).
[25] J. Ojeda-Castañeda, C.M. Gómez-Sarabia, Phot. Lett. Poland **7**, 14 (2015).


 