Photonic Band Structure of Fibonacci Superlattices with Metamaterials. Part 2: The Algebraic Method of Calculation for Absorptive Metamaterial Layers

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Abstract—We investigate the properties of the photonic band structure (PBS) of Fibonacci superlattices (FS) containing metamaterial absorptive layers. We study the properties of the PBS by the transfer matrix method (TMM). The energetic transmission coefficient for polarized light falling oblique on finite and binary FS placed in free space is calculated. The numerical results concerning transmission bands and energetic absorption coefficient are presented. The existence of fractal properties of PBS, and so called zero- $\bar{n}$ gaps are confirmed in "Xmas Trees"-like transmission spectra. The absorption effects are also discussed.

The real metamaterials absorb electromagnetic waves [1, 2], therefore the photonic band structure's (PBS) calculation of superlattices containing metamaterial slabs [3-11] should take this property into account. Moreover, physically existing superlattices are always finite. The calculations of PBS presented in Part 1, which are based on translational invariance and the Bloch theorem are a good approximation for finite, sufficiently long superlattices with negligible absorption.

In this short note (Part 2) we shall present: a) the algebraic method of PBS calculation for finite Fibonacci superlattices (FSs) based on transmission properties of the studied system, b) the preliminary results of numerical studies.

The studied systems are binary finite multilayers which are composed of isotropic and homogeneous **A** and **B** slabs with refractive indices, relative permittivities, permeabilities, thicknesses, which are given by $n_\mathbf{A}$, $\varepsilon_\mathbf{A}$, $\mu_\mathbf{A}$, $d_\mathbf{A}$, and $n_\mathbf{B}$, $\varepsilon_\mathbf{B}$, $\mu_\mathbf{B}$, $d_\mathbf{B}$, respectively. The order of a layer in the structure is defined by substitution rules: $S_0 = \mathbf{B}$, $S_1 = \mathbf{A}$, $S_L = S_{L-1} \bullet S_{L-2}$, where $L = 2, 3, \ldots$, denotes the generation number and "•" means concatenation.

In order to obtain PBS of absorptive superlattices' we propose to calculate energetic transmission (T), reflection (R) and absorption (A) coefficients. The transfer matrix method (TMM) [8-12] is applied to a finite multilayer, which is the unit cell of the infinite FS studied in Part 1.

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Let us denote each layer with index j, which takes values from 0 to $J + 1$, where J is the Fibonacci number defining the number of layers in a unit cell ($j = 0$ for the external medium "in" and $j = J + 1$ for the external medium "out").

The incident plane wave is described with the following parameters: free space wavelength λ (equivalent with frequency ω), polarization (s or p), and incidence angle θ_{in}.

In the TMM amplitudes of the electric field vectors of incident (E_{in}^{+}), transmitted (E_{out}^{+}) and reflected (E_{in}^{-}) waves are related by the following formula:

$$\begin{bmatrix} E_{\text{in}}^{+} \\ E_{\text{in}}^{-} \end{bmatrix} = \mathbf{D}_{\text{in},1}\mathbf{P}_1\mathbf{D}_{1,2}\cdot\ldots\cdot\mathbf{P}_J\mathbf{D}_{J,\text{out}}\begin{bmatrix} E_{\text{out}}^{+} \\ E_{\text{out}}^{-} \end{bmatrix} = \mathbf{\Gamma}\begin{bmatrix} E_{\text{out}}^{+} \\ E_{\text{out}}^{-} \end{bmatrix}, \quad (1)$$

where $E_{\text{out}}^{-} = 0$, and $\mathbf{\Gamma}$ is the 2×2 characteristic matrix of the layered system. The propagation matrices $\mathbf{P}_j$ are diagonal,

$$\mathbf{P}_j = \begin{bmatrix} \exp(\mathrm{i}\varphi_j) & 0 \\ 0 & \exp(-\mathrm{i}\varphi_j) \end{bmatrix}, \quad (2)$$

where $\varphi_j = d_j n_j (2\pi/\lambda) \cos\theta_j$. The matrix $\mathbf{D}_{j,j+1}$, called the transmission matrix from jth to $(j + 1)$st slab, takes the form:

$$\mathbf{D}_{j,j+1} = \frac{1}{t_{j,j+1}}\begin{bmatrix} 1 & r_{j,j+1} \\ r_{j,j+1} & 1 \end{bmatrix}, \quad (3)$$

where $t_{j,j+1}$ and $r_{j,j+1}$ are the Fresnel coefficients of transmission and reflection, which are polarization dependent.

Energetic transmission (T), reflection (R) and absorption (A) coefficients are given by:

$$T = \left|\frac{1}{\Gamma_{11}}\right|^2, \quad R = \left|\frac{\Gamma_{21}}{\Gamma_{11}}\right|^2, \quad A = 1 - (T + R). \quad (4)$$

We present the numerical results of calculation for FS with the following slabs' materials:

A – free space (i.e. right handed material, RHM)

B – metamaterial with material dispersion (dispersive, absorptive left handed material, LHM):

$$\varepsilon(\omega)=1-\frac{\omega_{pe}^2}{\omega^2+i\gamma_e\omega},\quad \mu(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_{pm}^2+i\gamma_m\omega},\quad (5)$$

where γ_e and γ_m are parameters responsible for absorption and ω_{pe} = 15.1 [10^{15} Hz], ω_{pm} = 2.39 [10^{15} Hz], F = 0.98 , are the same as in Part 1.

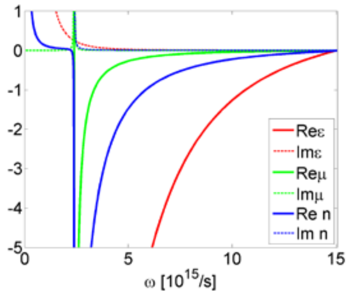

Fig. 2. Material dispersion of **B** (absorptive LHM) slab, permittivity – red, permeability – green, refractive index – blue, real parts in solid, imaginary parts in dashed lines; for ω_{pe}, ω_{pm}, F given in text and $\gamma_e = 10^{-3}\omega_{pe}$ and $\gamma_m = 10^{-3}\omega_{pm}$.

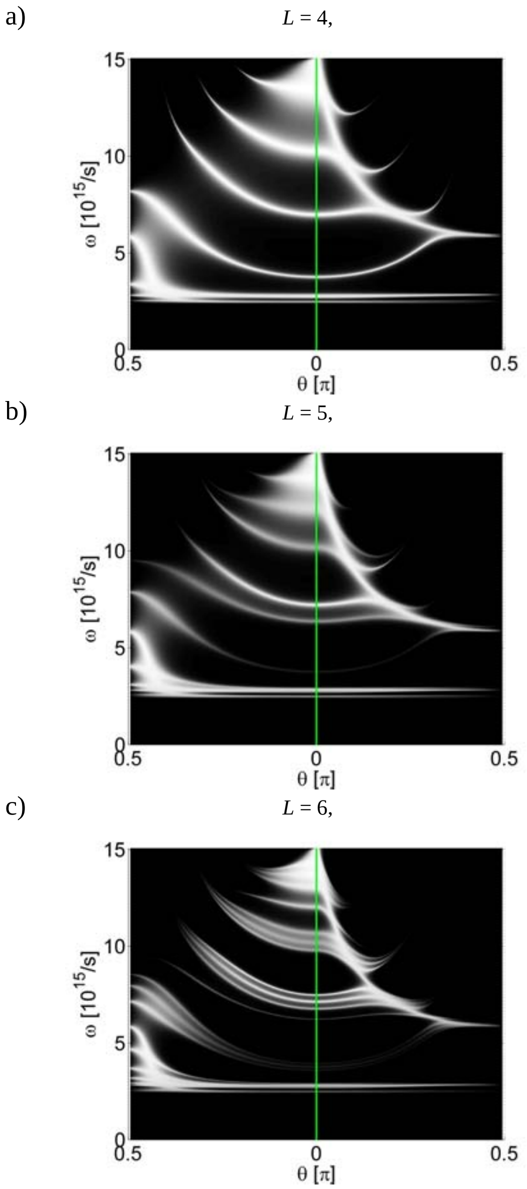

Fig. 1. Energetic transmission coefficient of FS for $\gamma_e = 10^{-3}\omega_{pe}$ and $\gamma_m = 10^{-3}\omega_{pm}$ and shown values of generation number L, polarization s and p, on the left and right side, respectively.

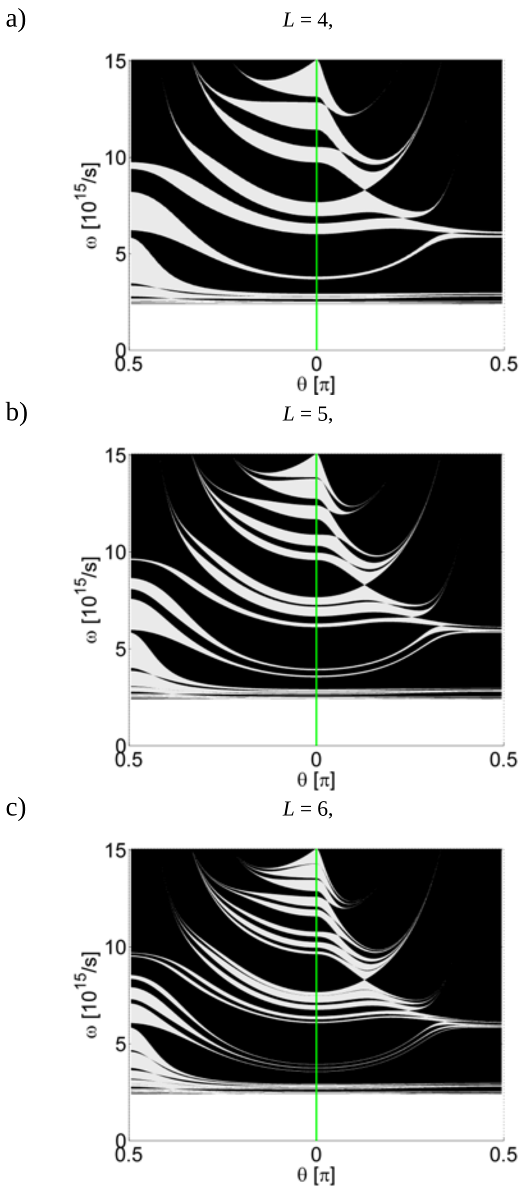

Fig. 3. PBS (cf. Part 1, Fig. 3.) of infinite FS at shown values of generation number L, polarization s and p, on left and right side, respectively.

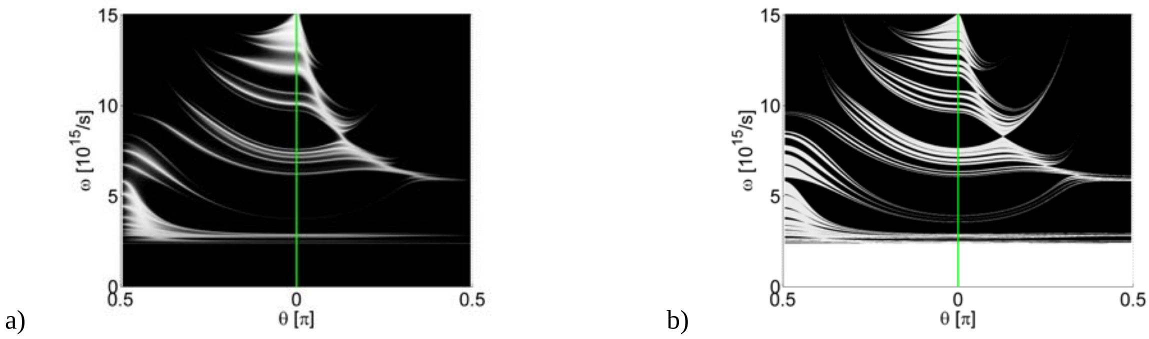

Fig. 4. Energetic transmission coefficient of FS and at generation number $L = 7$ (left), PBS (cf. Part 1, Fig. 3.) of infinite FS at generation number $L = 7$ (right), polarizations s and p, on left and right side of both plots, respectively.

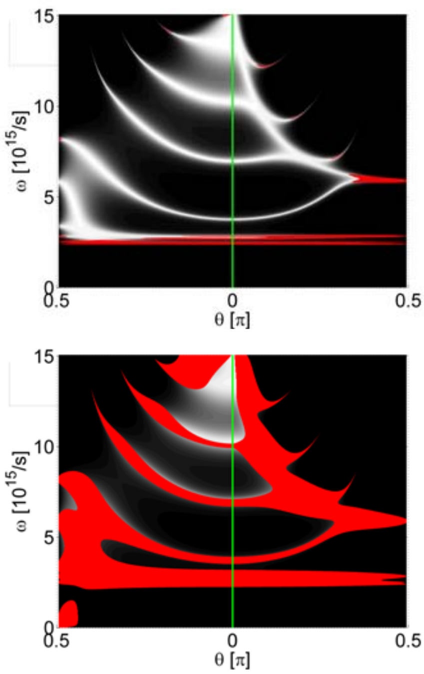

Fig. 5. Energetic transmission coefficient of FS unit cell for generation number $L = 4$ with areas of high absorption denoted; red regions indicate where $A > 0.1$; $\gamma_e = 10^{-3}\omega_{pe}$ and $\gamma_m = 10^{-3}\omega_{pm}$ (top), $\gamma_e = 10^{-2}\omega_{pe}$ and $\gamma_m = 10^{-2}\omega_{pm}$ (bottom); polarizations s and p, on left and right side of both plots, respectively.

The FS unit cell is placed in free space. The results are calculated in range $(0, \omega_{max})$, where $\omega_{max} = 15.069\cdot10^{15}$ Hz corresponds to the free space wavelength 125 nm. Figure 2 presents real and imaginary parts of $\varepsilon(\omega)$, $\mu(\omega)$ and $n(\omega)$. The thicknesses of layers are $d_{\mathbf{A}} = 125$ nm, $d_{\mathbf{B}} = 39.3$ nm, which gives the quarter-wave layers at a wavelength of 500 nm.

Concluding, we have presented the calculation method of a photonic band structure of finite FS. This method is based on the transfer matrix method [8-12]. Moreover, we want to point out that:

1. We have also observed the fractalization phenomenon (cf and compare Fig. 1 with Fig. 3 and Fig. 4) of absorptive Fibonacci superlattice photonic pass bands. The PBS of infinite structures has a higher number of pass bands, since in the single unit cell the number of **AB** and **BA** interfaces is too low to build up constructive interference. The calculation performed for a doubled and multiple cell shows convergence of transmission spectra to PBS.
2. Photonic band structure's calculation of finite superlattices with absorption, which cannot be neglected, requires applying the transfer matrix method presented here. Figure 5 shows calculations of energetic transmission spectra (which represent PBS). The areas of high absorption are indicated by the red color and pass bands of photonic band structure are hidden below.
3. Transmission spectra look like Xmas Trees s. Absorption effects are especially noticeable (Fig. 5.) on moustaches' endings (for relatively high values of incidence angle).
4. We have noticed zero-$\bar{n}$ gap areas; according to [6] absorption does not destroy this effect, although in those regions energy losses are remarkably large (cf. Fig. 5.).

Finally, let us note that the proposed approach can be used for computer designing of microoptical devices (filters, Bragg and omnidirectional reflectors, superlenses) containing periodic, aperiodic and random multilayers with metamaterials and for studies of light tunneling.

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