# Analysis of spontaneous emission in a 1D photonic crystal with effective resonator model

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**Abstract**—This letter discusses the application of effective resonator model as a tool for analysis of spontaneous emission in one-dimensional photonic crystals built of a finite number of layers and with arbitrary defects of layer's width or refractive index. It is pointed out that quantities defined in the model allow analyzing the properties of the structure without calculating field distributions.

Photonic crystals are periodically arranged dielectric materials which possess photonic band gaps – frequency ranges for which propagation of light inside the structure is forbidden. They have been actively investigated since the works of Yablonovitch [1], who suggested using them to suppress spontaneous emission, and John [2], who pointed out possible localization of light if a proper defect was introduced. One-dimensional photonic crystals (1DPC) are the simplest structures of this kind. They are a particularly good subject of studies, because they are of practical importance (they can be used to build LEDs with increased efficiency [3], DFB or VCSEL lasers [4,5], filters [6,7] and other devices) and are characterized by the same features as more complex photonic crystal structures, but unlike them, they can be modelled analytically. Periodical structures can be described with the help of the Floquet-Bloch theorem, as in e.g. [8]. However, real structures are never perfectly periodical, first of all because their dimensions are finite, but also because they contain intentionally introduced defects. Therefore, in general, they should be considered a specific case of a planar multilayer waveguide (see Fig. 1, where $j$ – index of the layer, $n_{(j)}$ – refractive index). These structures can be modelled with a generalized Carniglia-Mandel model [9,10], in which the modes of electromagnetic field are obtained as combinations of plane waves, coming from a plane wave incident on the structure from the outside (*incoming modes*) or propagating away from it (*outgoing modes*) [11], which determines amplitudes of all the other plane waves in layers of the structure (they can be calculated e.g. with the translation matrix method [12, 13]) – see Fig. 2 (a), (b). This approach is correct and widely used, but it does not naturally fit the description of spontaneous emission from within the structure, because modes fixed by waves from the outside of the structure do not explicitly show the properties of the region surrounding the emitter – to investigate them it is necessary to manipulate and analyze the obtained field distributions. A much more relevant approach is provided by the effective resonator model [14-17], in which the construction of modes starts with plane waves *inside* the same layer the emitter is situated in (Fig. 2c). In this model, a parameter, so-called *mode spectrum* is defined, which clearly and explicitly shows the properties of the layer interesting for a designer of the structure.

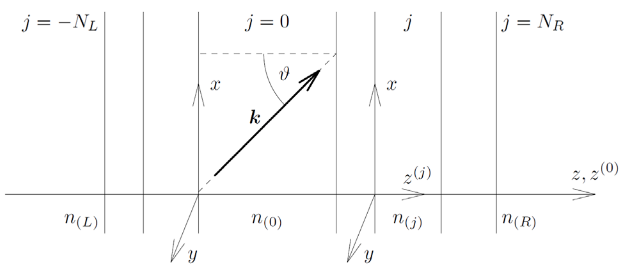


Fig. 1. Sketch of a finite one-dimensional photonic crystal (1DPC) with arbitrary defects.

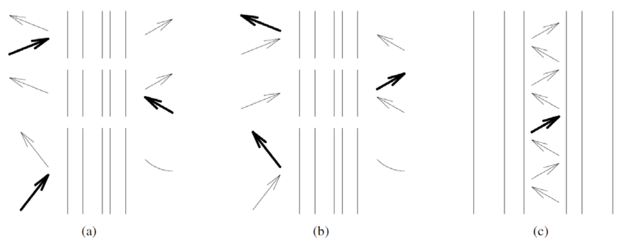


Fig. 2. Modes in different models of 1DPC: (a) – incoming modes, (b) – outgoing modes, (c) – effective resonator model. Bold arrows – waves fixing a mode; evanescent waves indicated by arcs in (a), (b).

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The translation matrix method allows for an easy calculation of the reflection coefficient of a stack of dielectric layers, thus, a layer (for which we can agree to put $j$=0) can be considered as a resonator, with two mirrors: "left" and "right", reflection coefficients of which correspond to those of dielectric stacks on both sides of the layer (Fig. 3). Suppose that such resonator is excited with a plane wave with wave vector $\boldsymbol{k}$ and polarization $\varepsilon$. To satisfy boundary conditions, reflected waves have to appear and the resulting electromagnetic field in the resonator (layer) is their superposition [14, 16]. It can be written as:

$$\psi_\varepsilon(\boldsymbol{k}) = \rho_\varepsilon(\boldsymbol{k})\left[\boldsymbol{e}_{k\varepsilon}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}) + \xi_\varepsilon^*(\boldsymbol{k})\boldsymbol{e}_{k_*\varepsilon}\exp(i\boldsymbol{k}_*\cdot\boldsymbol{r})\right], \quad (1)$$

where $\boldsymbol{k}_*=k_x\boldsymbol{e}_x+k_y\boldsymbol{e}_y-k_z\boldsymbol{e}_z$ denotes vector $\boldsymbol{k}$ with changed sign of $k_z$ component (after $xy$-plane reflection), the quantity

$$\rho_\varepsilon(\boldsymbol{k}) = \frac{1}{8\pi^3}\frac{1-|r_L r_R|^2}{|1-r_L r_R\exp(2ik_z L_z)|^2}, \quad (2)$$

is called *mode spectrum*, the *coupling coefficient*:

$$\xi_\varepsilon(\boldsymbol{k}) = \frac{r_R^*\left(1-|r_L|^2\right)\exp(-2ik_z L_z) + r_L\left(1-|r_R|^2\right)}{1-|r_L r_R|^2} \quad (3)$$

(for clarity, dependence of reflection coefficients on the wave vector and polarization is not explicitly denoted), $\boldsymbol{e}_{k\varepsilon}$ is the polarization versor and $L_z$ – width of the layer. Mode spectrum is the quantity which contains information about the layer's properties and, in particular, characterizes the rate of spontaneous emission from an atom situated in the layer [18]. Thus, the investigation of how the structure affects spontaneous emission can be based on this quantity.

The same construction of modes can be performed for any other layer, with $j=J$ (without loss of generality we assume $J>0$, for simplicity), leading to its mode spectrum $\rho_\varepsilon'(\boldsymbol{k}')$ and coupling coefficient $\xi_\varepsilon'(\boldsymbol{k}')$. If $\boldsymbol{k}'$ is a wave vector of a plane wave, which could be matched with a plane wave with a wave vector $\boldsymbol{k}$ in the 0th layer to satisfy field continuity conditions, then using these conditions it is easy to show that the mode spectrum and coupling coefficient of modes in both layers are bound by the following expressions:

$$\rho_\varepsilon'(\boldsymbol{k}') = \rho_\varepsilon(\boldsymbol{k})\left[1+\frac{2\left|(\mathbf{m}_{J,0})_{12}\right|^2}{\det\mathbf{m}_{J,0}}\left(1+\mathrm{Re}\left\{\frac{(\mathbf{m}_{J,0})_{11}}{(\mathbf{m}_{J,0})_{12}}\xi_\varepsilon(\boldsymbol{k})\right\}\right)\right], \quad (4)$$

and

$$\xi_\varepsilon'(\boldsymbol{k}') = \frac{\rho_\varepsilon(\boldsymbol{k})}{\rho_\varepsilon'(\boldsymbol{k}')}\frac{(\mathbf{m}_{J,0})_{11}^2\xi_\varepsilon(\boldsymbol{k}) + (\mathbf{m}_{J,0})_{12}^2\xi_\varepsilon^*(\boldsymbol{k}) + 2(\mathbf{m}_{J,0})_{11}(\mathbf{m}_{J,0})_{12}}{\det\mathbf{m}_{J,0}}, \quad (5)$$

where $\mathbf{m}_{J,0}$ is the matrix relating amplitudes of plane waves in both layers so that they would satisfy the continuity conditions [12, 16, 17]. Therefore, analysis of properties of the 1DPC structure (the modification of a spontaneous emission rate in particular) with the help of the effective resonator model does not at all require calculating actual field distributions, what greatly simplifies this task.

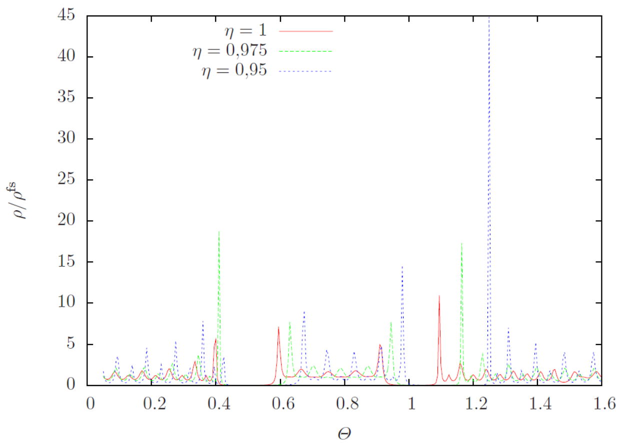


Fig. 3. Exemplary mode spectrum in a 1DPC for various angles of incidence.

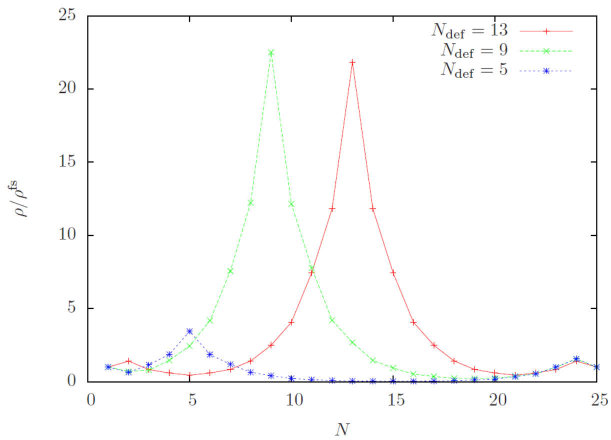


Fig. 4. Exemplary mode spectrum in a defected 1DPC in various layers for defect of width in elementary cell $N_{def}$.

Exemplary results obtained with effective resonator model are presented in Figs. 3 and 4. Figure 3 shows the mode spectrum in a 1DPC in function of normalized frequency $\Theta$ (value $\Theta$=0.5 corresponds to Bragg frequency) for various $\eta=\cos\theta$, where $\theta$ – angle of incidence. On this plot, the mode spectrum is normalized to free space value $\rho^{fs}=1/8\pi^3$, directly indicating modification of spontaneous emission (with respect to free space) by the structure. It can be seen that the mode spectrum shows locations of photonic band gaps and modes "preferred" by the structure. Other results, which show how localization of light by a defect changes with its position, are depicted in Fig. 4, in which $N$ indexes elementary cells, starting from the edge of the structure, and the mode spectrum has been calculated for the layer with higher refractive index in each elementary cell. For a defect situated in $N_{def}^{th}$ elementary cell of the structure, the amplitude of electromagnetic field, which is characterized by the mode spectrum, is the highest in the defected layer. This shows that spontaneous emission is enhanced in the defected layer or nearby, but in other regions of the structure it is not. Moreover, the localization and enhancement depend on how close to the edge the defect has been placed. Such conclusion could not be so easily drawn with the other mentioned models.

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