Tunable hyperbolic apodizer

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Abstract—We unveil the use of two grey level masks for controlling continuously the attenuation of a spatial filter, which has an exponentially decreasing hyperbolic profile. We present analytical expressions that help to visualize the influence of our proposed grey level masks for reducing the side-lobes of a 1-D point spread function.

In optical spectroscopy for describing the use of non-uniform windows, the most commonly used notation is optical apodization. The word apodization was coined by Pierre Jaquinot for denoting the reduction of the side-lobes of the impulse response, or *point spread function* (PSF) [1].

When an amplitude mask is used for reducing the side-lobes, then as a secondary effect, the amplitude mask broadens the main lobe of the PSF. Some experts may not be aware of the following analogy; which exploits McCutchen theorem [2-6].

If one considers the axial point spread function (axial PSF) of an optical system, then reducing the side-lobes and widening the central lobe is useful for broadening the axial impulse response. And therefore, an apodizing mask with radial symmetry is useful for increasing the focal depth of an optical system [7-10].

We note that amplitude masks, with moderate absorption, also find useful applications for reducing the spurious oscillations, *in the modulation transfer functions* (MTF), which are generated by employing phase masks that diminish the impact of focus error [11-13].

Here our goal is to present the use of two grey level masks for controlling the damping factor of an apodizer, which has an exponentially decreasing hyperbolic profile. We unveil analytical expressions that helps to visualize the influence of our proposed masks for reducing the side-lobes of the PSF.

For our present discussion we consider the telecentric, optical processor depicted in Fig. 1. At the Fraunhofer plane, of the optical setup in Fig. 1, we have two identical 1-D, amplitude masks that work as a pair. The pupil aperture, along the horizontal axis, is 2Ω. Hence, if the Greek letter μ is a spatial frequency variable; the cut-off spatial frequency is Ω.

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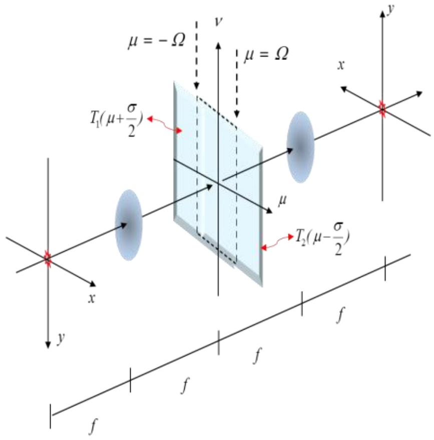

Fig. 1. Optical setup under consideration.

At the Fraunhofer plane, of the optical setup in Fig. 1, the pupil aperture is represented by the function rect(μ/2Ω). This function is equal to unity if |μ| ≤ Ω; otherwise the rectangular function is equal to zero. For avoiding any mechanical vignetting, we consider that the masks are larger than the pupil aperture. For this purpose the lateral extension of the masks is represented as rect(μ/6Ω). In mathematical terms, the amplitude transmittance of the 1-D mask is

$$T_1(\mu) = T_2(\mu) = \exp\left[-a\cosh\left(2\pi\frac{\mu}{\Omega}\right)\right]\mathrm{rect}(\frac{\mu}{6\Omega}). \qquad (1)$$

After we place the two mask in closed contact, we introduce a lateral displacement σ, where σ < Ω. Then, the overall amplitude transmittance of the two masks can be expressed as

$$\begin{aligned} Q(\mu;\sigma) &= T_1(\mu+\frac{\sigma}{2})T_2(\mu-\frac{\sigma}{2})\,\mathrm{rect}(\frac{\mu}{2\Omega}) \\ &= \exp\left\{\left[-2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right]\cosh(2\pi\frac{\mu}{\Omega})\right\}\mathrm{rect}(\frac{\mu}{2\Omega}). \end{aligned} \qquad (2)$$

It is apparent from Eq. (2) that the overall amplitude transmittance is a real, positive, even function. In Fig. 2

we plot the result in Eq. (2) for a=0.02, and for several values of σ/Ω = 0, 0.1, 0.2, 0.3, 0.4 and 0.5.

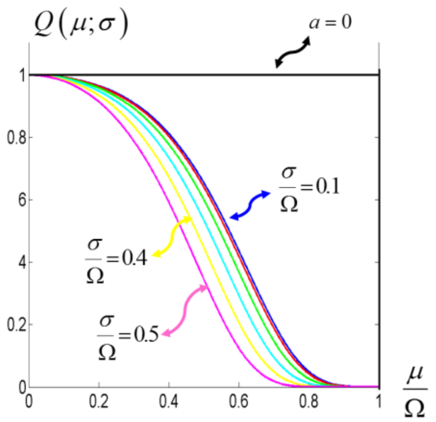

Fig. 2. Amplitude transmittance as a function of the spatial frequency μ, for several values of σ/Ω.

From Eq. (2), and from Fig. 2, we claim that by laterally displacing the two masks, one can tune the damping factor of the apodizer, while preserving the hyperbolic profile. Next, we analyse the influence of this apodizer on the PSF side-lobes.

One can obtain the irradiance PSF, by taking the square modulus of the inverse Fourier transform, of Eq. (2). In mathematical terms, the irradiance PSF is

$$\left|q\left(x;\sigma\right)\right|^2=\left|\int_{-\infty}^{\infty}\mathrm{Q}(\mu;\sigma)\exp\left(i2\pi x\mu\right)d\mu\right|^2. \qquad (3)$$

We employ conventional software tools for evaluating the fast Fourier transform in Eq. (3); and in this manner we obtain the irradiance PSF shown in Fig. 3. From the numerical results in Fig. 3, we claim the following. As one increases the lateral displacement (between the two grey level masks), the irradiance PSF has attenuated side-lobes. At the same time, the main lobe of the PSF has a broader width. Furthermore, one can analyse the influence of the proposed masks, as follows.

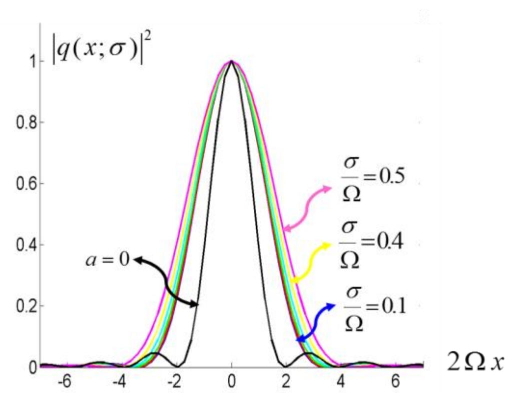

Fig. 3. Irradiance distributions associated to the PSF of the proposed mask, for several values of lateral displacement.

First, we recognize that Eq. (2) can also be rewritten (using the procedure in the Appendix) as

$$\begin{aligned}&\exp\left\{\left[-2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right]\cosh(2\pi\frac{\mu}{\Omega})\right\}\\&=I_0\left[2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right]+\\&+2\sum_{n=1}^{\infty}\left(-1\right)^n I_n\left[2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right]\cosh(2\pi\frac{\mu}{\Omega}\mathrm{n}).\end{aligned} \qquad (4)$$

For the sake of simplicity, we have omitted the rect(μ/Ω) function at both sides of Eq. (4). Next, we recognize that the first term in Eq. (4) is practically equal to unity for a = 0.02, and for the values of σ/Ω that appear in Fig. 2.

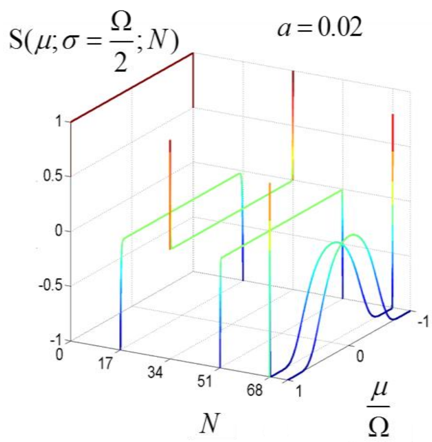

Fig. 4. The influence of the maximum number of terms N on S(μ; σ; N)

For understanding the role of the remaining terms in Eq. (4), we evaluate numerically the following expression

$$\mathrm{S}(\mu;\sigma;N)=1+2\sum_{n=1}^{N}\left(-1\right)^{n}I_{n}\left[2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right]\cosh(2\pi\frac{\mu}{\Omega}\mathrm{n}). \quad (5)$$

In Fig. 4 we show the numerical results that are obtained by evaluating Eq. (5), for a = 0.02 and for σ/Ω = 0.5, and by considering that the maximum number of terms is N = 17, 34, 51, and 64. The series becomes stable for N = 74. From Fig. 4 we note that as one increases the number of terms, one starts to obtain the same profile as in Fig. 2.
Furthermore, from Eq. (4), we note that the amplitude PSF can be written explicitly as

$$\mathrm{q}(\mathrm{x};\sigma)=I_{0}\left[2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right]\ \mathrm{sinc}\left(\Omega x\right)+ +2\sum_{n=1}^{\infty}\left(-1\right)^{n}I_{n}\left[2a\cosh\left(\pi\frac{\sigma}{\Omega}\right)\right] \int_{-\Omega}^{\Omega}\cosh(2\pi\frac{\mu}{\Omega}\mathrm{n})\exp\left(i2\pi x\mu\right)\mathrm{d}\mu. \quad (6)$$

We recognize that the first term in equation 6 is almost equal to the sinc function. Hence, the remaining terms are responsible of reducing the side-lobes.

In conclusion, we have proposed the use of two identical grey level masks for controlling continuously the damping factor of an optical apodizer; which has an exponentially decreasing hyperbolic profile. We have shown that by introducing a lateral displacement between the grey level masks, one can tune continuously the damping factor, while preserving the hyperbolic profile. We have presented analytical expressions that help to visualize the influence of our proposed apodizer for reducing the side-lobes of the PSF; while broadening the width of the main lobe.

Appendix: We start by using the well-known generating function of the Bessel function

$$\exp\left[\frac{z}{2}(t-\frac{1}{t})\right]=\sum_{n=-\infty}^{\infty}J_{n}(z)\,t^{n} \quad (A1)$$

Then, we make the following change of variables

$$z=i\,\beta,\quad t=i\exp\left(-2\pi\frac{\mu}{\Omega}\right). \quad (A2)$$

In Eq. (A2), as in the main text, i =√-1; μ is the spatial frequency variable; and Ω denotes the cut-off spatial frequency. By substituting Eq. (A2) in Eq. (A1) we obtain

$$\exp\left[-\beta\cosh(2\pi\frac{\mu}{\Omega})\right]=\sum_{n=-\infty}^{\infty}J_{n}(\mathrm{i}\,\beta)\exp(-2\pi n\frac{\mu}{\Omega}). \quad (A3)$$

Next, we recognize the following two relationships of the modified Bessel functions $I_n(\beta) = J_n(i\beta) = (i)^n I_n(\beta)$; $I_n(\beta) = I_{-n}(\beta)$; see for example reference [14]. By taking into account these relationships, we obtain

$$\exp\left[-\beta\cosh(2\pi\frac{\mu}{\Omega})\right]=I_{0}(\beta)+2\sum_{n=1}^{\infty}\left(-1\right)^{n}I_{n}(\beta)\cosh(2\pi n\frac{\mu}{\Omega}). \quad (A4)$$

The result in Eq. (A4) is used as Eq. (4), in the main text, with

$$\beta=2a\cosh\left(\pi\frac{\sigma}{\Omega}\right). \quad (A5)$$

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References

[1] P. Jacquinot, B. Roizen-Dossier, *Apodisation* in Prog. Opt. 3, E. Wolf editor (North Holland, 1964) pp. 29-186.
[2] C.W. McCutchen, J. Opt. Soc. Am. **54**, 240 (1964).
[3] H. Arsenault, A. Boivin, J. Appl. Phys. **38**, 3988 (1967).
[4] C.W. McCutchen, J. Opt. Soc. Am. A **8**, 868 (1991).
[5] J.E.A. Landgrave, L.R. Berriel-Valdos, J. Opt. Soc. Am. A **14**, 2962 (1997).
[6] A.W. Lohmann, J. Ojeda-Castañeda, P. Rodríguez-Montero, Opt. Comm. **108**, 42 (1994).
[7] J. Ojeda-Castafieda, L.R. Berriel-Valdos, E. Montes, Appl. Opt. **26**, 2770 (1987).
[8] J. Ojeda-Castañeda, A. Díaz, Appl. Opt. **27**, 4163 (1988).
[9] J. Ojeda-Castañeda, L.R. Berriel-Valdos, Opt. Lett. **13**, 183 (1988).
[10] J. Ojeda-Castañeda, L.R. Berriel-Valdos, Appl. Opt. **29**, 994 (1990).
[11] J. Ojeda-Castañeda, E. Yépez-Vidal, E. García-Almanza, Phot. Lett. Poland **2**, 162 (2010).
[12] J. Ojeda-Castaneda, E. Yépez-Vidal, E. García-Almanza, C.M. Gómez-Sarabia, Phot. Lett. Poland **4**, 115 (2012).
[13] J. Ojeda-Castaneda, S. Ledesma, C.M. Gómez-Sarabia, Phot. Lett. Poland **5**, 23 (2013).
[14] W.W. Bell, *Special functions for scientists and engineers* (Van Nostrand, London 1968).

