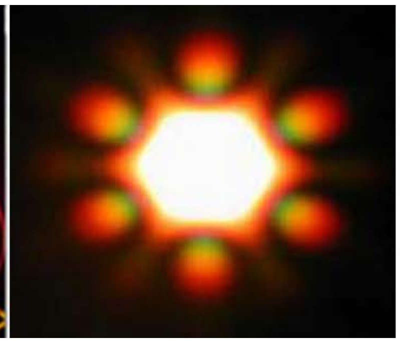
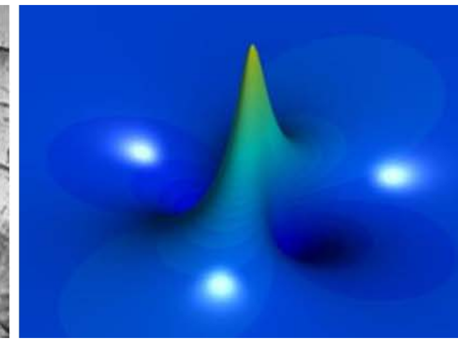
1000 years of optics – 50 years of solitons

John Dudley
CNRS Institut FEMTO-ST
Université de Franche-Comté
Besançon, France

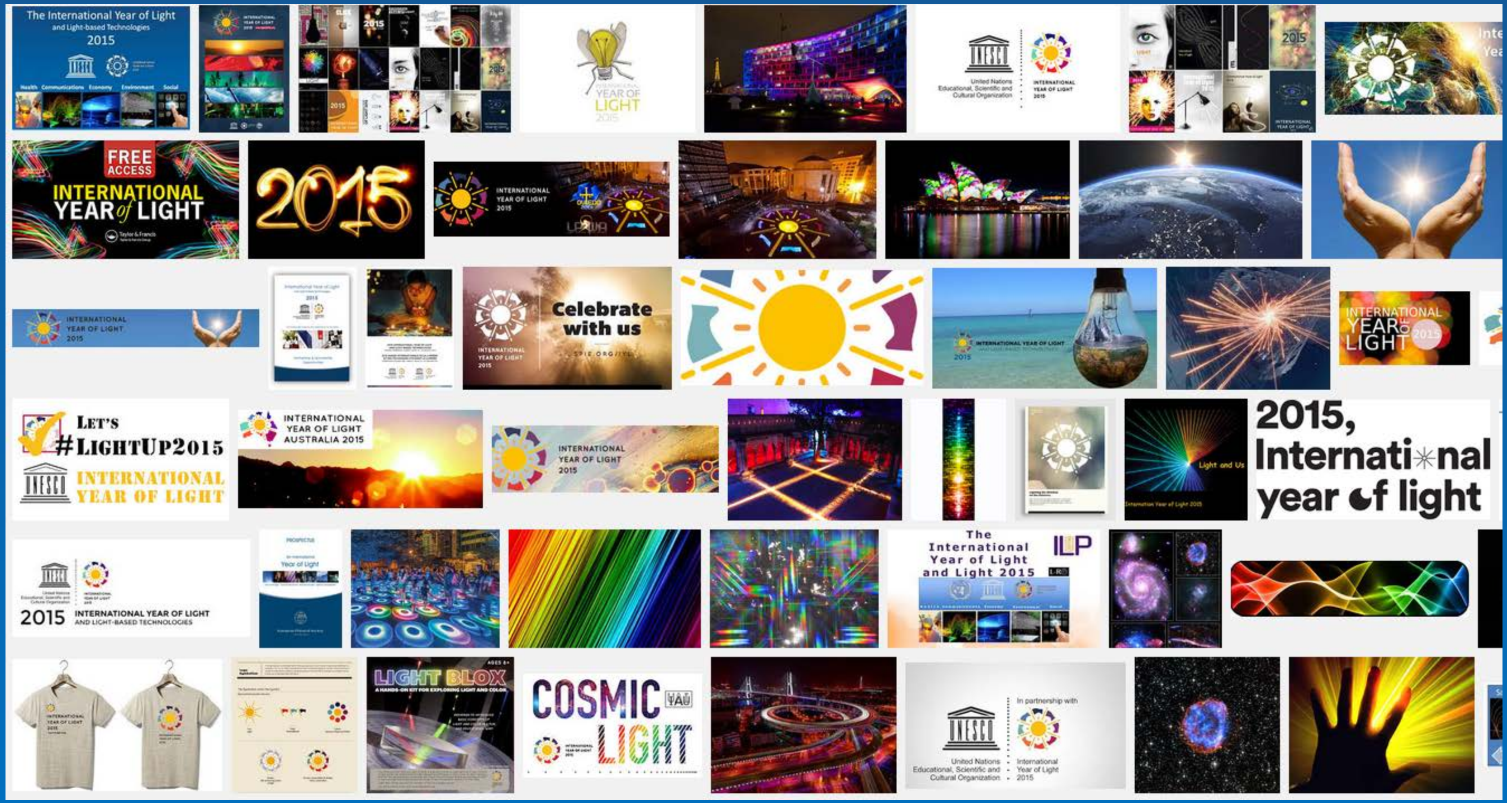
femto-st SCIENCES & TECHNOLOGIES
cnrs
erc

Frederic Dias, Goery Genty, Benjamin Wetzel,
Thomas Godin, Shanti Toenger, Pierre-Ambroise Lacourt and many others…

International Year of Light 2015

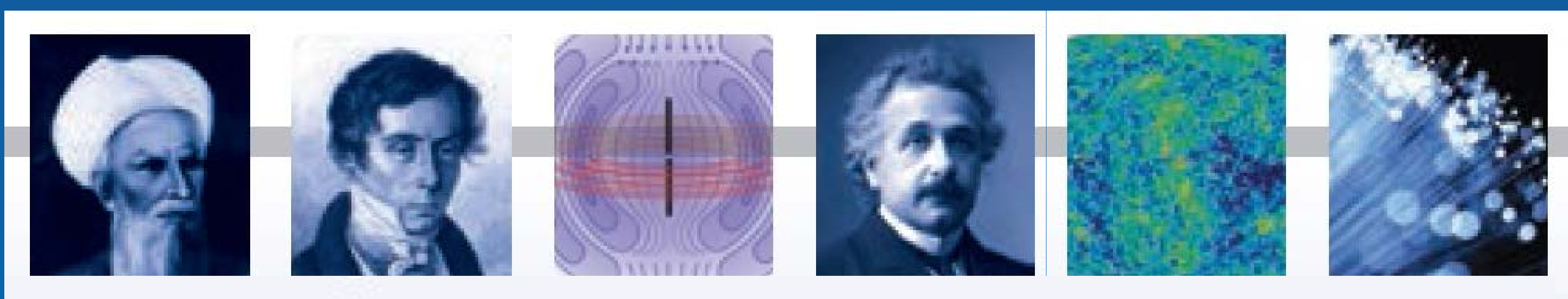
3rd Symposium of the Photonics Society of Poland
Warsaw, 8-9 April 2015

The International Year of Light and Light-based Technologies 2015



The International Year of Light and Light-based Technologies 2015

1015 1815 1865 1915 1965



Al-Haytham Fresnel Maxwell Einstein Penzias & Wilson, Kao

"Light sciences are a cross-cutting discipline in the 21st century"

Ban Ki Moon

But there was one that we could not get in …

VOLUME 15, NUMBER 6 PHYSICAL REVIEW LETTERS 9 AUGUST 1965

INTERACTION OF "SOLITONS" IN A COLLISIONLESS PLASMA AND THE RECURRENCE OF INITIAL STATES

N. J. Zabusky

Bell Telephone Laboratories, Whippany, New Jersey

and

M. D. Kruskal

Princeton University Plasma Physics Laboratory, Princeton, New Jersey

(Received 3 May 1965)

We have observed unusual nonlinear interactions among "solitary-wave pulses" propagating in nonlinear dispersive media. These phenomena were observed in the numerical solutions of the Korteweg-deVries equation

$$u_t + uu_x + \delta^2 u_{xxx} = 0. \qquad (1)$$

This equation can be used to describe the one-

or "soliton" begins to move uniformly at a rate (relative to the background value of u from which the pulse rises) which is linearly proportional to its amplitude. Thus, the solitons spread apart. Because of the periodicity, two or more solitons eventually overlap spatially and interact nonlinearly. Shortly after the interaction, they reappear virtually unaffected in size or shape. In other words, solitons "pass through"

This Talk

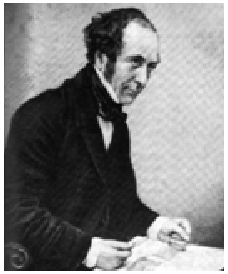
Solitons

Solitons in nonlinear optics

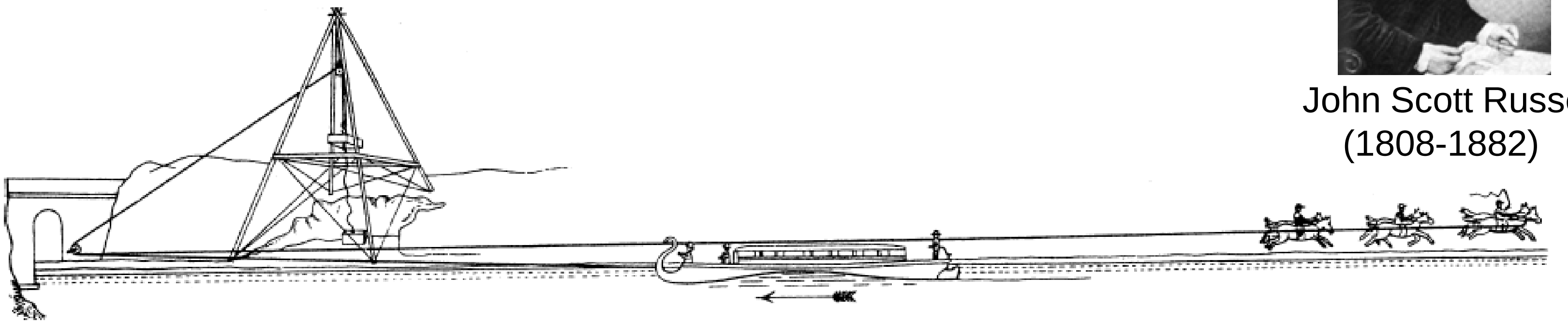
Insights into extreme events in other systems

The soliton was discovered in 1834

When studying barge speed and hull design
on the Union Canal Edinburgh …



John Scott Russell
(1808-1882)

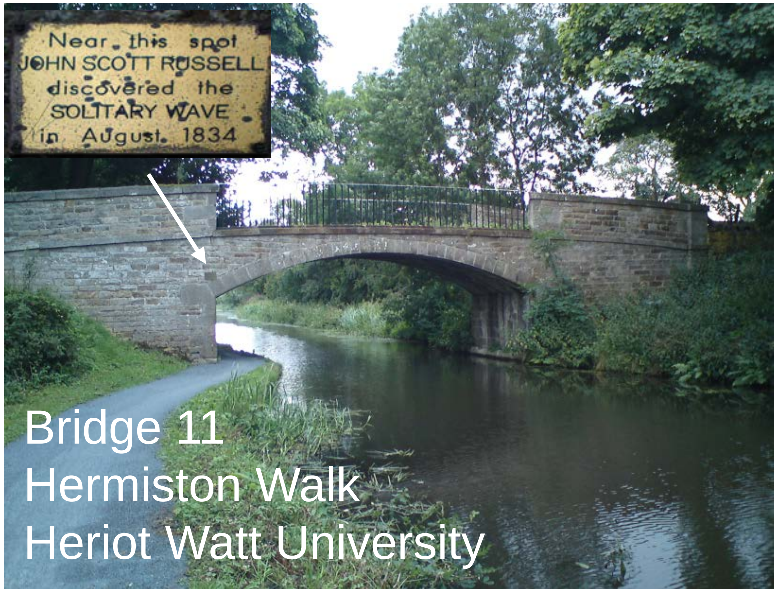


1834: "The happiest day of my life"

"the boat suddenly stopped –
not so the mass of water in the
channel which it had put in motion

…

a large, solitary, progressive wave"



O. Darrigol, The Spirited Horse, the Engineer, and the Mathematician … Arch. Hist. Exact Sci. **58**, 21-95 (2003)

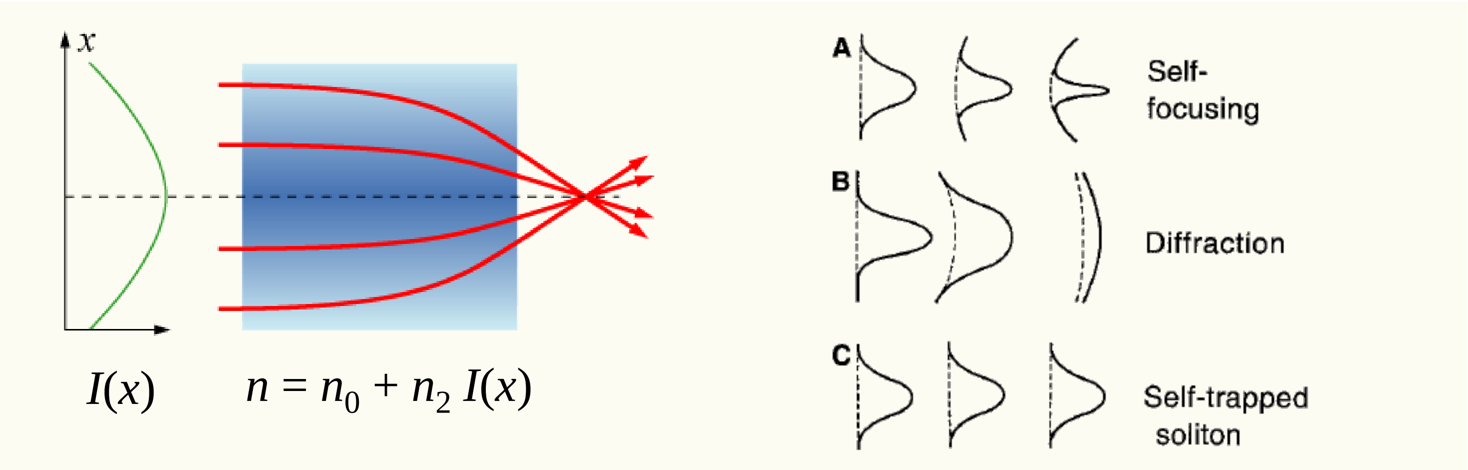
Why this is unusual – a modern example with light

Waves usually disperse (diffract) with propagation

Definition of a soliton

A physical structure that does not change during propagation

Nonlinear self-compression (self-focussing) balances the usual linear tendency of a wave to disperse (or diffract)



A soliton timeline

Confirmation and Theory

1865: Henri Bazin
1871: Joseph Boussinesq
1895: Korteweg – de Vries

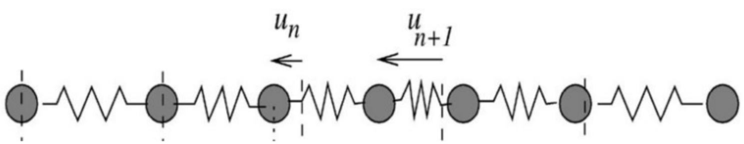
$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + \gamma u\frac{\partial u}{\partial x} + \varepsilon\frac{\partial^3 u}{\partial x^3} = 0$$

$$c = \sqrt{gd} \qquad \varepsilon = c(h^2/6 - T/2\rho g)$$

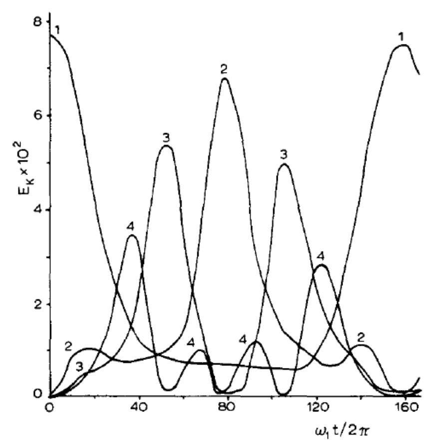
$$\gamma = 3c/2h$$

Solitons in Nonlinear Science

1953: Fermi-Pasta-Ulam recurrence



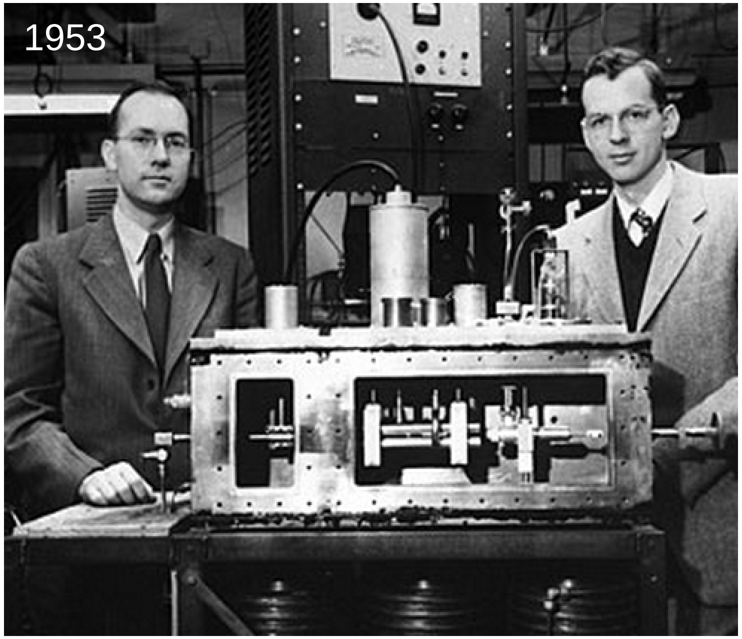
1965: Zabusky & Kruskal “solitons”
1967: Lorentz
Deterministic nonperiodic flow



1972: Zakharov & Shabat
Nonlinear Schrödinger equation
solitons

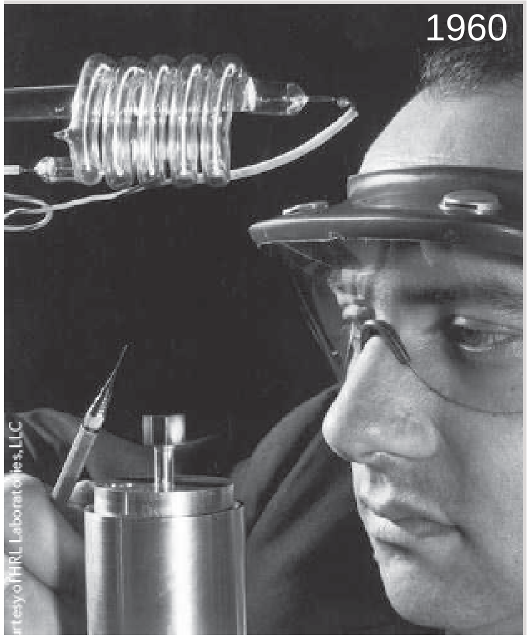
The laser enabled study of solitons using light

Maser



Townes Gordon

Laser



Maiman

Optical spatial solitons

VOLUME 13, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1964

SELF-TRAPPING OF OPTICAL BEAMS*

R. Y. Chiao, E. Garmire, and C. H. Townes
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 1 September 1964)

We shall discuss here conditions under which an electromagnetic beam can produce its own dielectric waveguide and propagate without spreading. This may occur in materials whose dielectric constant increases with field intensity, but which are quite homogeneous in the absence of the electromagnetic wave. Such self-trapping in dielectric waveguide modes appears to be possible in intense laser beams, and to produce marked optical and physical effects.

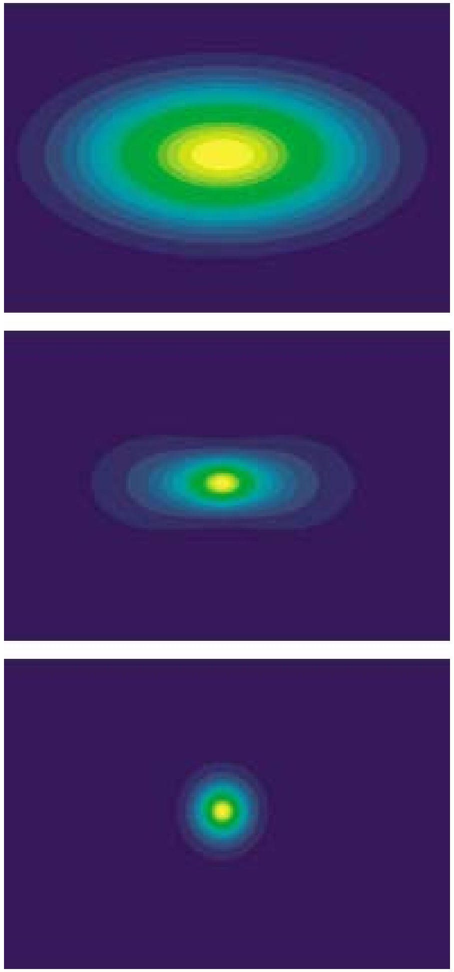
.....

*Work supported in part by the National Aeronautics and Space Administration under Research Grant No. NsG-330, and in part by the U. S. Air Force Cambridge Research Laboratories, Office of Aerospace Research, under Contract No. AF19(628)-4011.
[1]M. Hercher, J. Opt. Soc. Am. 54, 563 (1964).

Charles H. Townes
(1915 - Jan 27 2015)

Townes et al.
Pub Astro. Soc Pac.
123 1370-1373 (2011)

Optical spatial solitons



Observation of filamentation in a Kerr medium

1964: Hercher

Self-trapping of optical beams

1964: Chiao, Garmire and Townes

Observation of spatial optical solitons

1974: Ashkin and Bjorkholm
1985: Barthelemy, Froehly
1992: Photorefractive solitons
2004: Self similar collapse & the Townes profile

Low-loss optical waveguide development

Reliable techniques for fabricating small-core waveguides yielded the birth of optical fibre communications

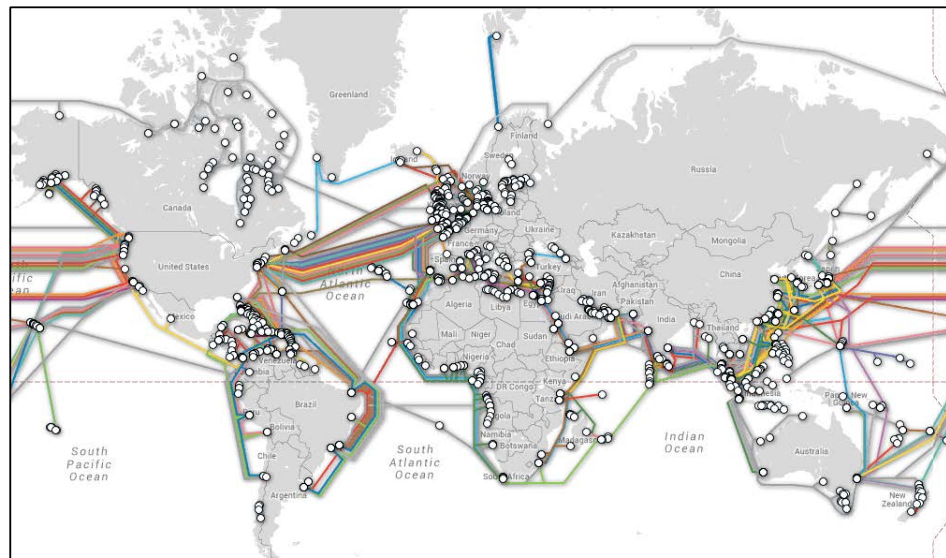
International Year of Light 2015

PROC. IEE, Vol. 113, No. 7, JULY 1966

Dielectric-fibre surface waveguides for optical frequencies

K. C. Kao, B.Sc.(Eng.), Ph.D., A.M.I.E.E., and G. A. Hockham, B.Sc.(Eng.),

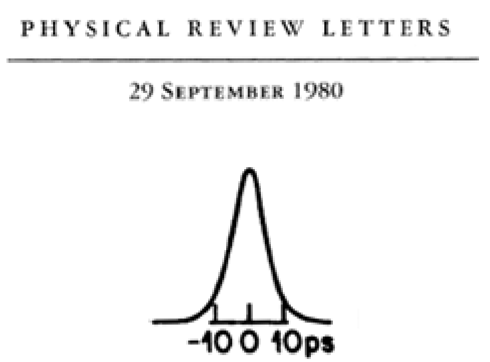
The Nobel Prize in Physics 2009



Optical temporal solitons

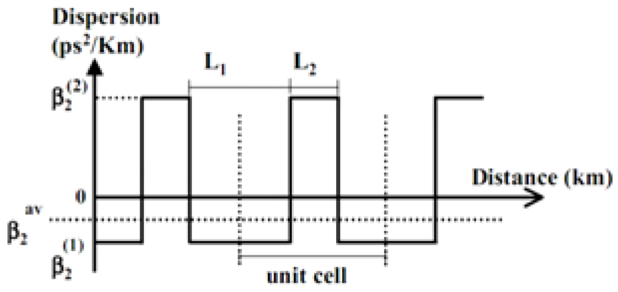
Theory of solitons in optical fibres

1973: Hasegawa & Tappert



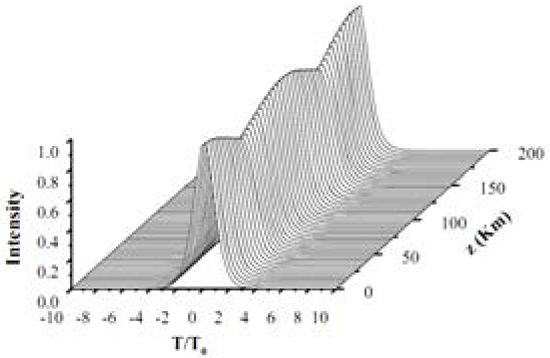
Laboratory demonstrations

1980: Mollenauer, Stolen, Gordon (bright)
1987: Emplit et al. (dark)



EDFAs, Dispersion control

1989: Desurvire, Payne
1990's: Doran, Mollenauer



Today: nonlinearity is "managed"

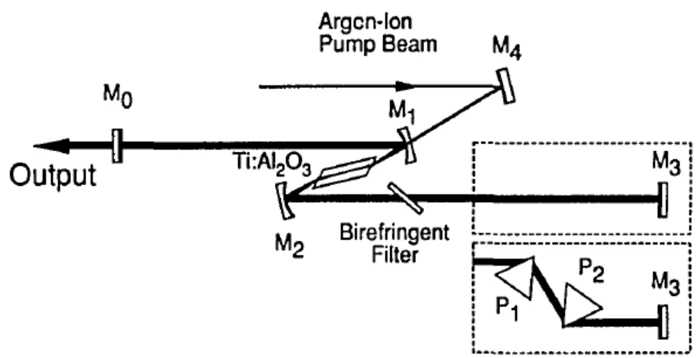
Solitons in modelocked lasers …

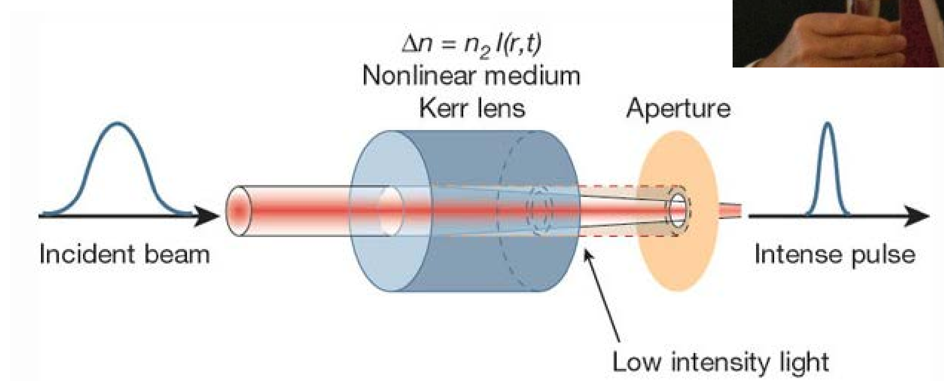
A well-known example is the self-modelocked Ti:Sapphire laser

42 OPTICS LETTERS / Vol. 16, No. 1 / January 1, 1991

60-fsec pulse generation from a self-mode-locked Ti:sapphire laser

D. E. Spence, P. N. Kean, and W. Sibbett





Temporal soliton dynamics

dispersion management

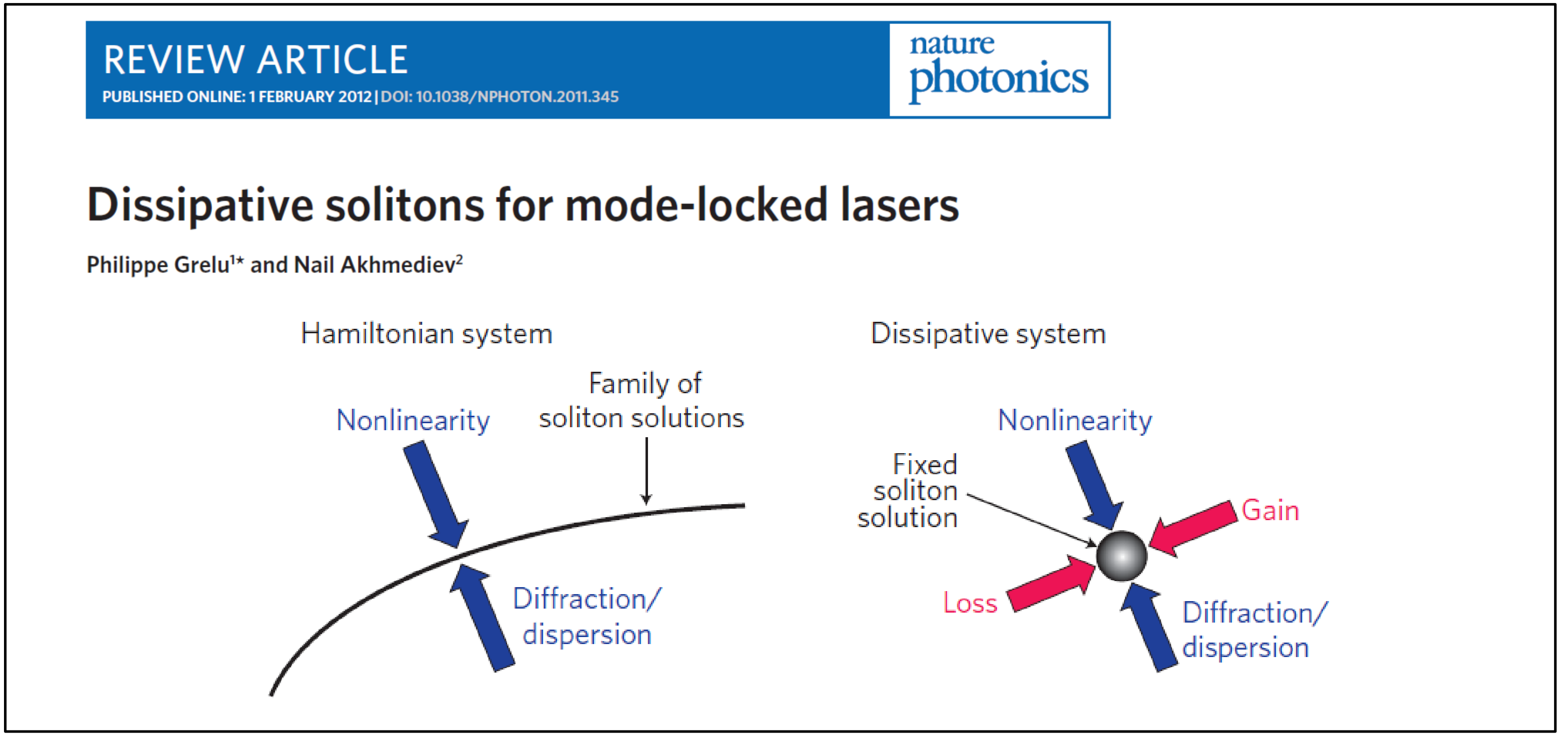
Spatial soliton dynamics

diffraction management

W Sibbett *et al.* The development and application of femtosecond laser systems
Optics Express Focus Issue on Modular Ultrafast Lasers **20** 6989-7001 (2012)

Relaxing the definition of a “soliton” opens new theory

The **dissipative soliton** concept expands the mathematical definition of solitons to include energy exchange, opening up new applications


REVIEW ARTICLE

PUBLISHED ONLINE: 1 FEBRUARY 2012 | DOI: 10.1038/NPHOTON.2011.345

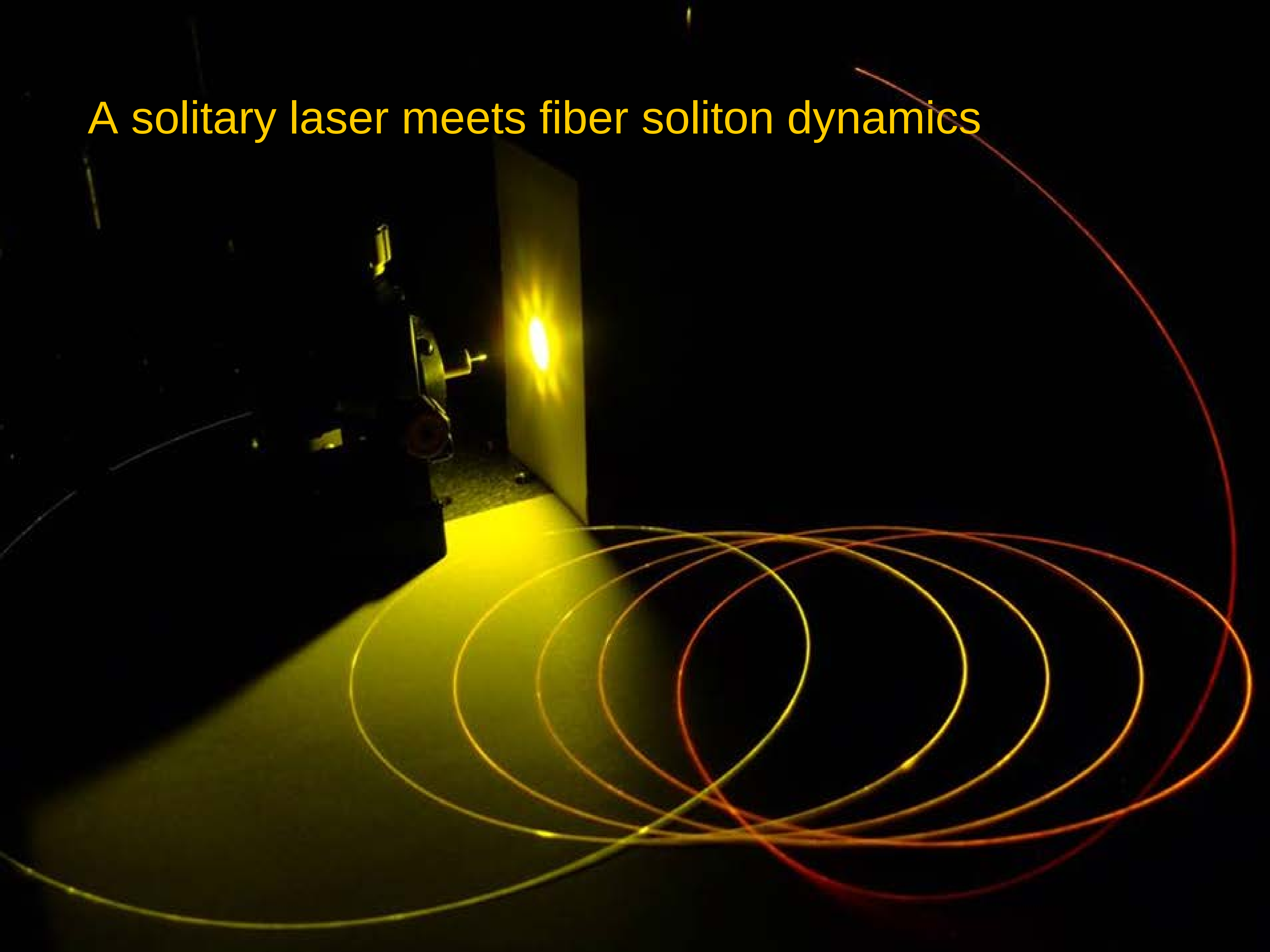
nature photonics

Dissipative solitons for mode-locked lasers

Philippe Grelu[1*] and Nail Akhmediev[2]

In this sense, nearly everything can be considered as a soliton!

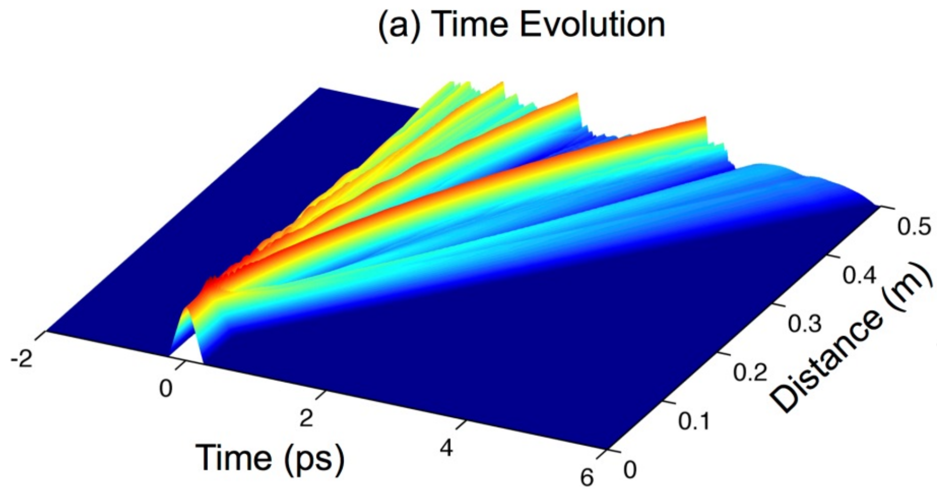
A solitary laser meets fiber soliton dynamics

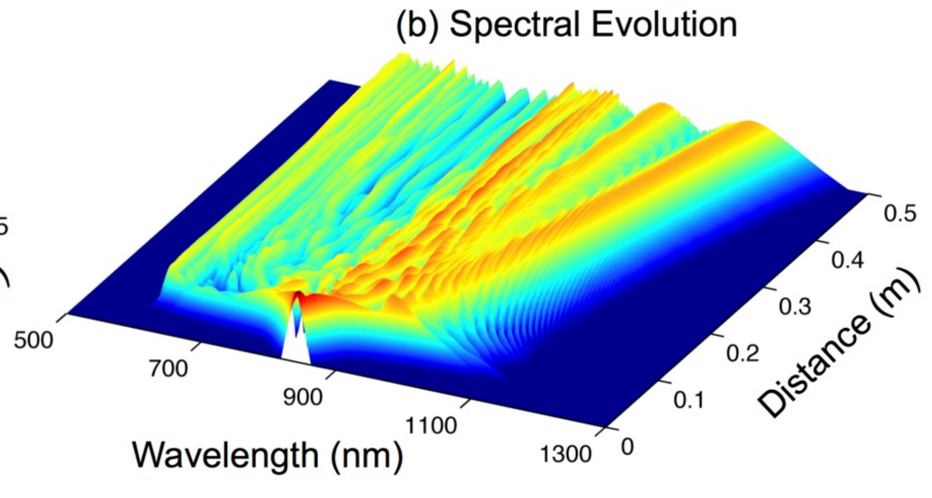


Understanding the supercontinuum dynamics

Modelled using a generalized nonlinear Schrödinger equation (NLSE)

$$\underbrace{\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k>2}\frac{\mathrm{i}^{k+1}}{k!}\beta_k\frac{\partial^k A}{\partial T^k}}_{\text{Linear dispersion}} = i\gamma\left(\underbrace{1 + \mathrm{i}\tau_{\mathrm{shock}}\frac{\partial}{\partial T}}_{\text{Self-steepening}}\right)\left(\underbrace{A(z,t)\int_{-\infty}^{+\infty} R(T')|A(z,T-T')|^2 dT'}_{\text{SPM, FWM, Raman}}\right)$$

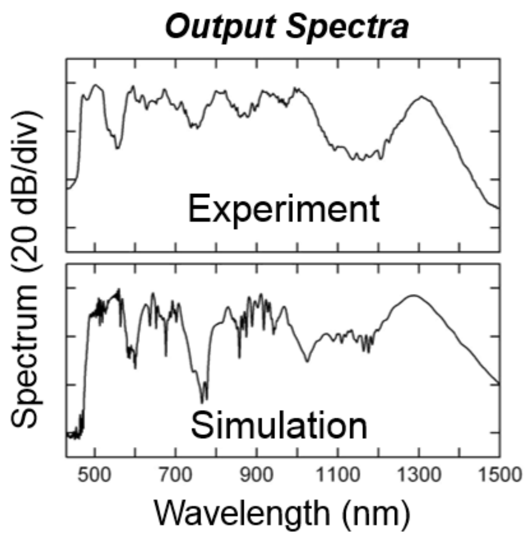


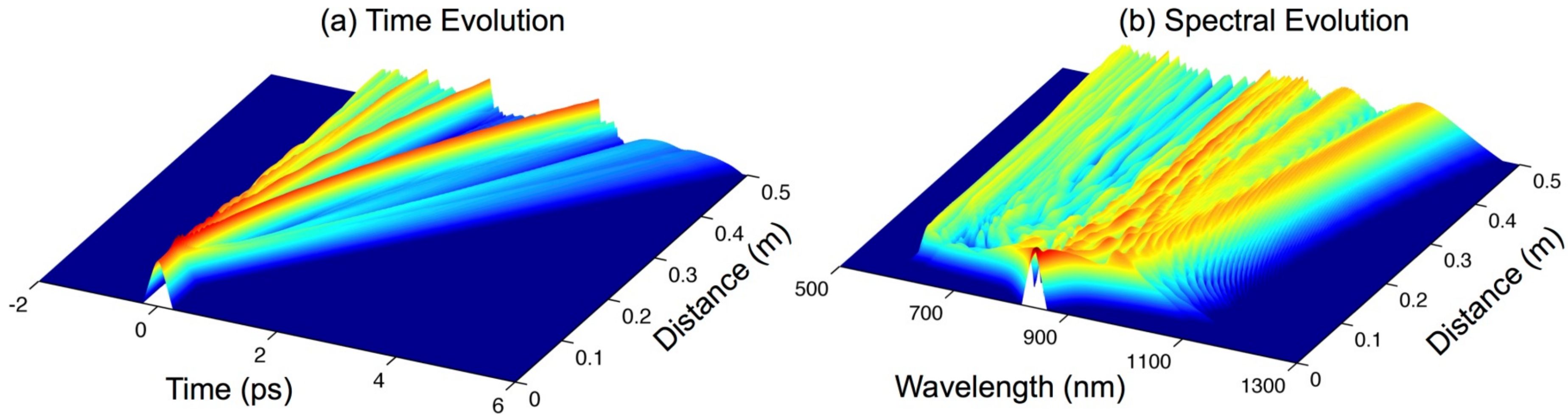


Dudley, Genty, Coen, RMP **78** 1135 (2006)

Understanding the supercontinuum dynamics

1. Input pulse is a high-order soliton
2. Perturbation due to higher-order dispersion
3. Fission into fundamental solitons
4. Raman self-frequency shift (RED)
5. Dispersive wave generation (BLUE)

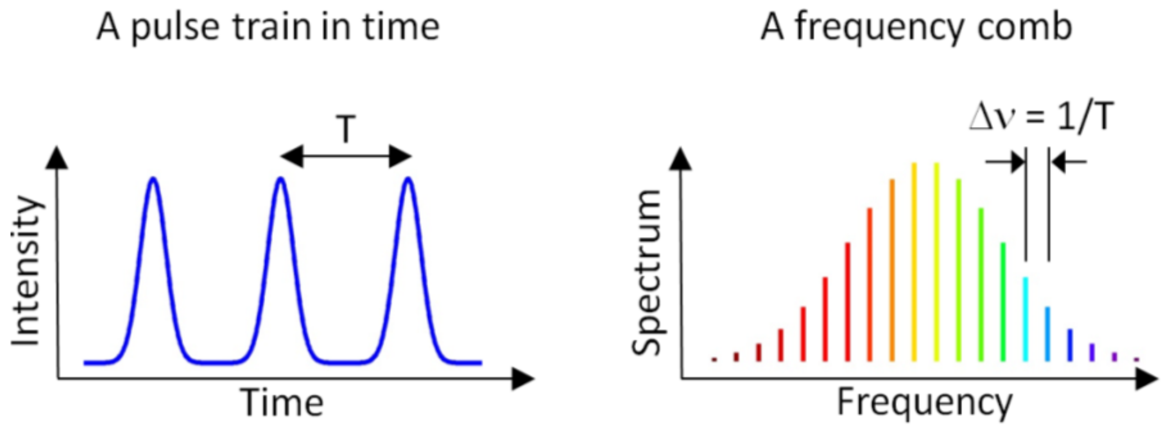




Dudley, Genty, Coen, RMP **78** 1135 (2006)

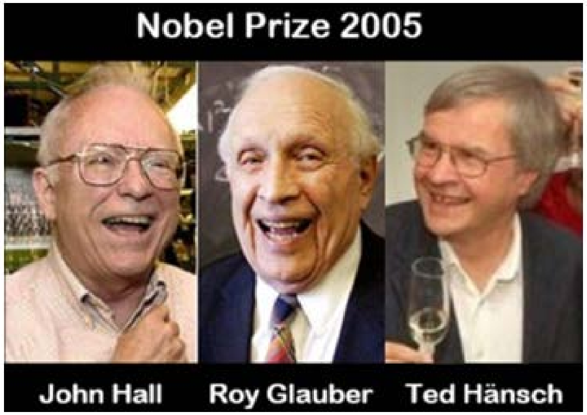
Immediate interest and impact

Basic description of ultrashort pulses



An octave-spanning spectrum allows comb position to be readily stabilized

We can bridge the gap between a known optical frequency locked to definition of the second and any optical frequency



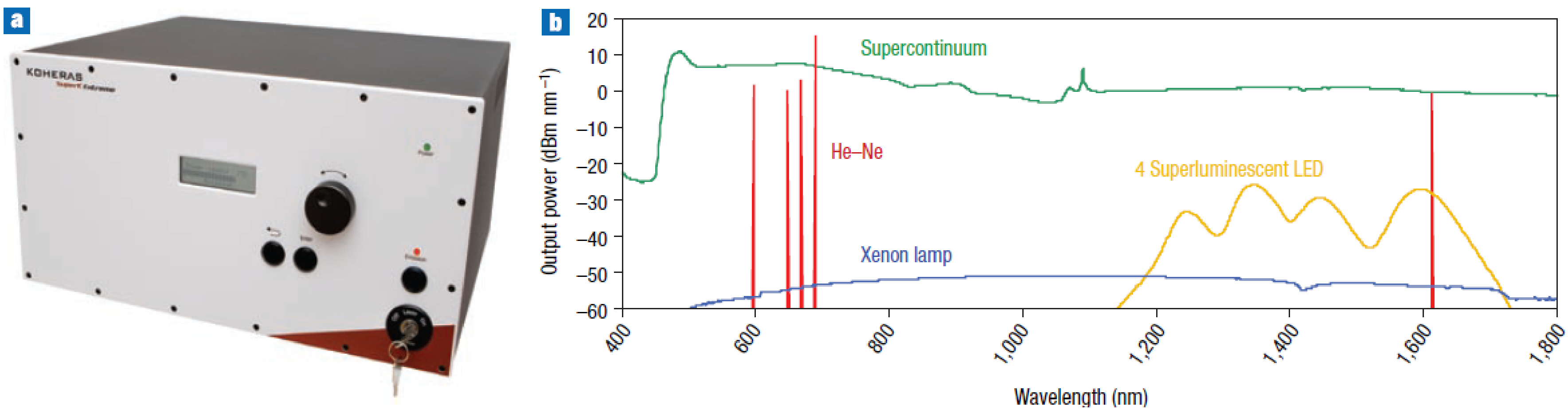
Immediate interest and impact

nature photonics | VOL 2 | JANUARY 2008 | www.nature.com/naturephotonics

INDUSTRY PERSPECTIVE | TECHNOLOGY FOCUS

METROLOGY

Broad as a lamp, bright as a laser



Molecular fingerprinting
Human breath analysis
Above 10 μm

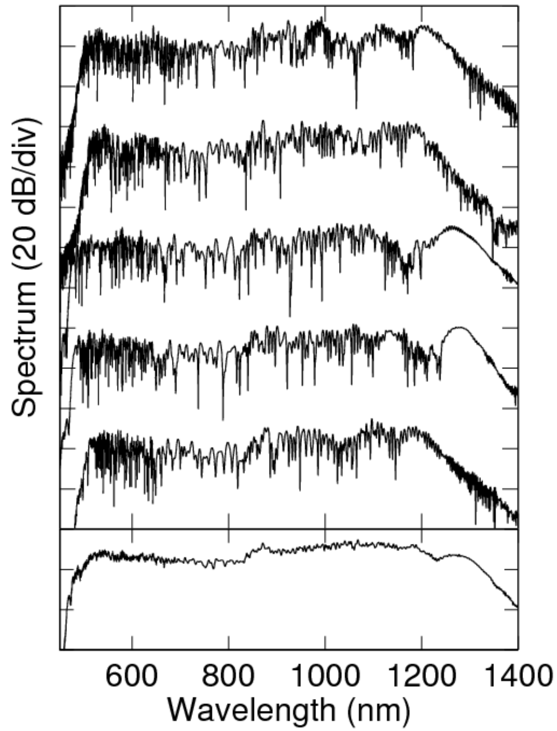
S. Diddams *et al.* Nature **445**, 627 (2007)
M. J. Thorpe *et al.* Opt. Express **16**, 2387 (2008)
C. R. Petersen et al. Nature Photon. **8**, 830 (2014)

Unstable solitons in supercontinuum generation

Modelling reveals that the supercontinuum can be highly unstable

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A - \sum_{k\geq 2}\frac{\mathrm{i}^{k+1}}{k!}\beta_k\frac{\partial^k A}{\partial T^k} = i\gamma\left(1 + \mathrm{i}\tau_{\mathrm{shock}}\frac{\partial}{\partial T}\right)\left(A(z,t)\int_{-\infty}^{+\infty}R(T')|A(z,T-T')|^2 dT'\right)$$

Stochastic simulations



5 individual realisations, identical apart from quantum noise

Successive pulses from a laser pulse train generate significantly different spectra

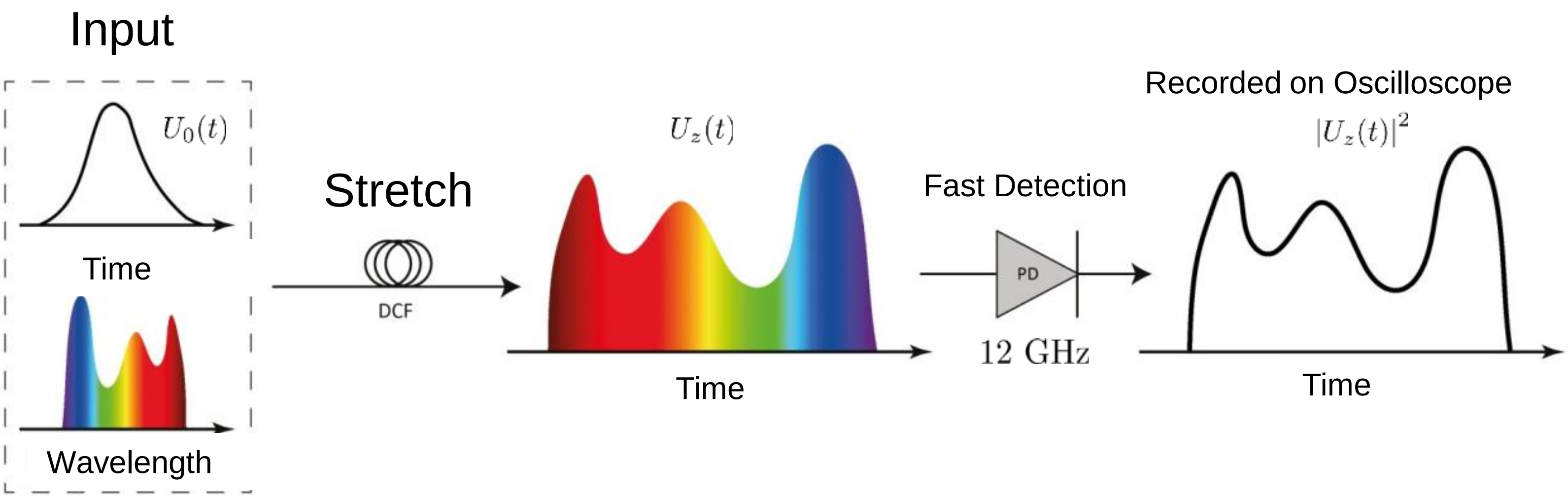
We measure an artificially smooth spectrum, but the noise is still present

J. M. Dudley, G. Genty, S. Coen, Rev. Mod. Phys. **78** 1135 (2006)

Measuring soliton noise on a shot-to-shot basis

New measurement techniques provide new opportunities to study noisy processes in nonlinear optics via direct statistical characterisation

Dispersive Time Stretching



$$U_z(t) \approx \widetilde{U_0}(t/\beta_{2s}z)\,\exp\left(\frac{-it^2}{2\beta_{2s}z}\right)$$

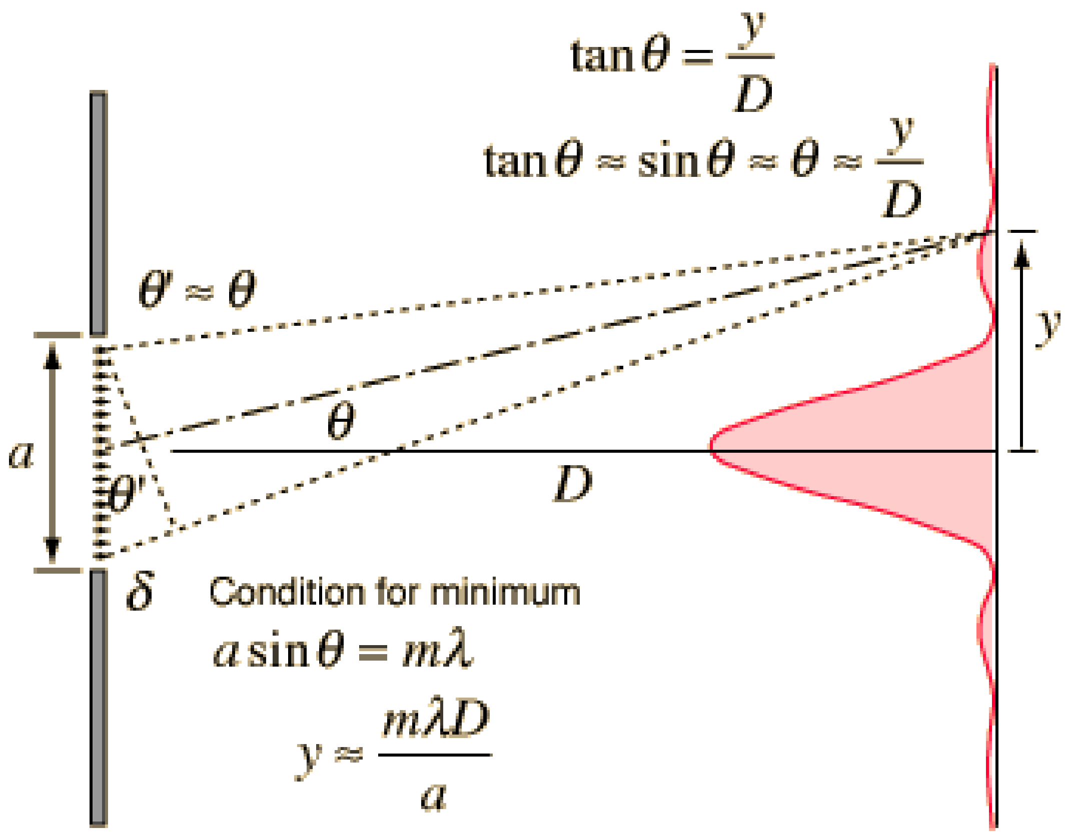
$$|U_z(t)|^2 \approx \left|\widetilde{U_0}(\omega)\right|^2 \quad \omega = 2\pi\nu = \frac{t}{\beta_{2s}z}$$

P.M. Duffieux *« L'intégrale de Fourier et ses applications à l'optique »* (1946)

G. V. Grigoryan et al., *OPN* **11** (2000)

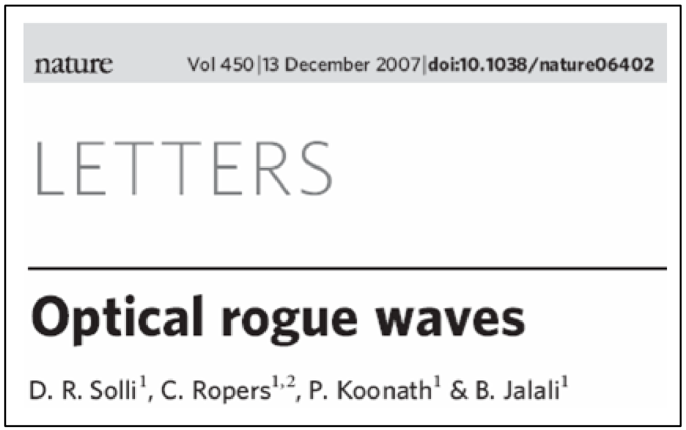
Far field diffraction and Fourier optics

This is the time-domain equivalent of the fact that the far field diffraction pattern of an aperture is the Fourier transform of the diffracting mask



Rare soliton events are optical rogue waves

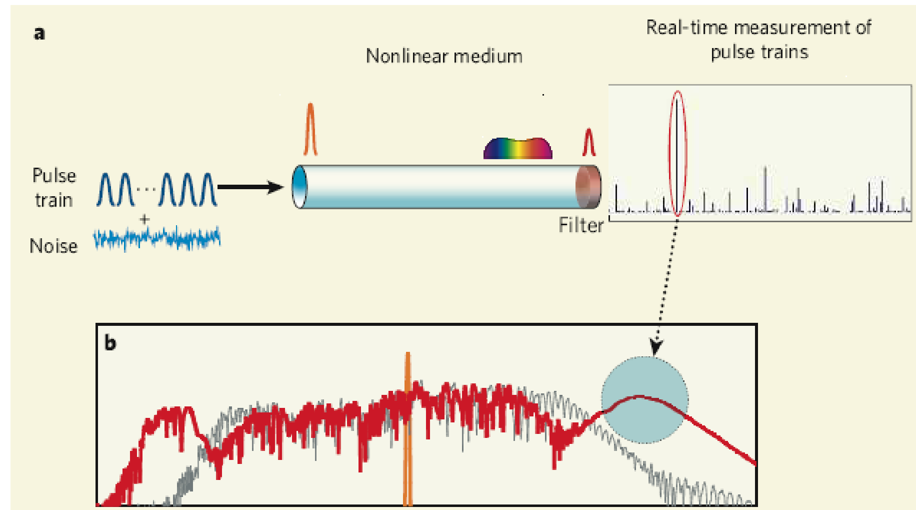
Experiments reveal that these instabilities yield long-tailed statistics


nature Vol 450 | 13 December 2007 | doi:10.1038/nature06402

LETTERS

Optical rogue waves

D. R. Solli[1], C. Ropers[1,2], P. Koonath[1] & B. Jalali[1]

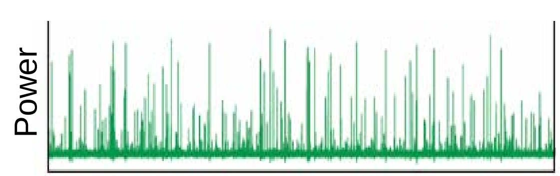


Also:

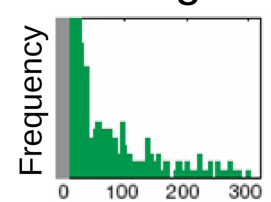
Wetzel et al.
Scientific Reports **2**, 882 (2012)

Godin et al.
Optics Express **21** 18452-18460 (2013)

Time series



Histogram



Rogue waves on the ocean

Long tails in the wave height distribution are claimed to be rogue wave signature

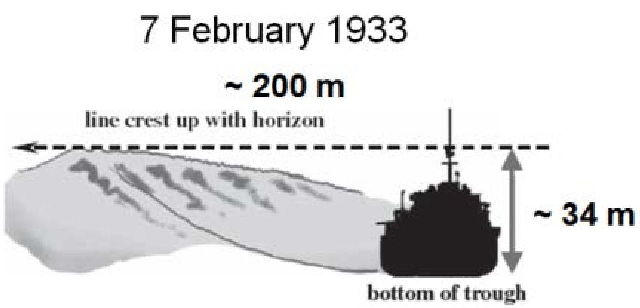
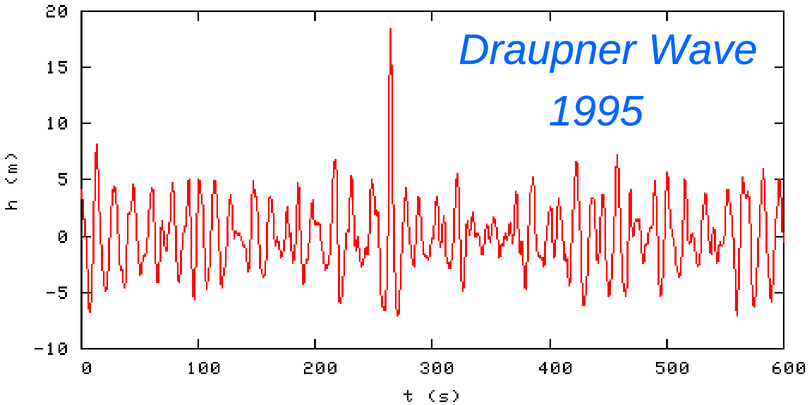


Fig. I.2 Observation of the highest reported wave by the crew members of "Ramapo" (Dennis and Wolff 1996)



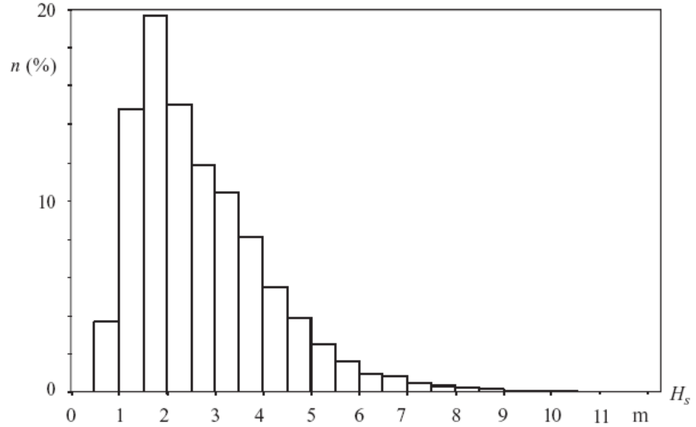
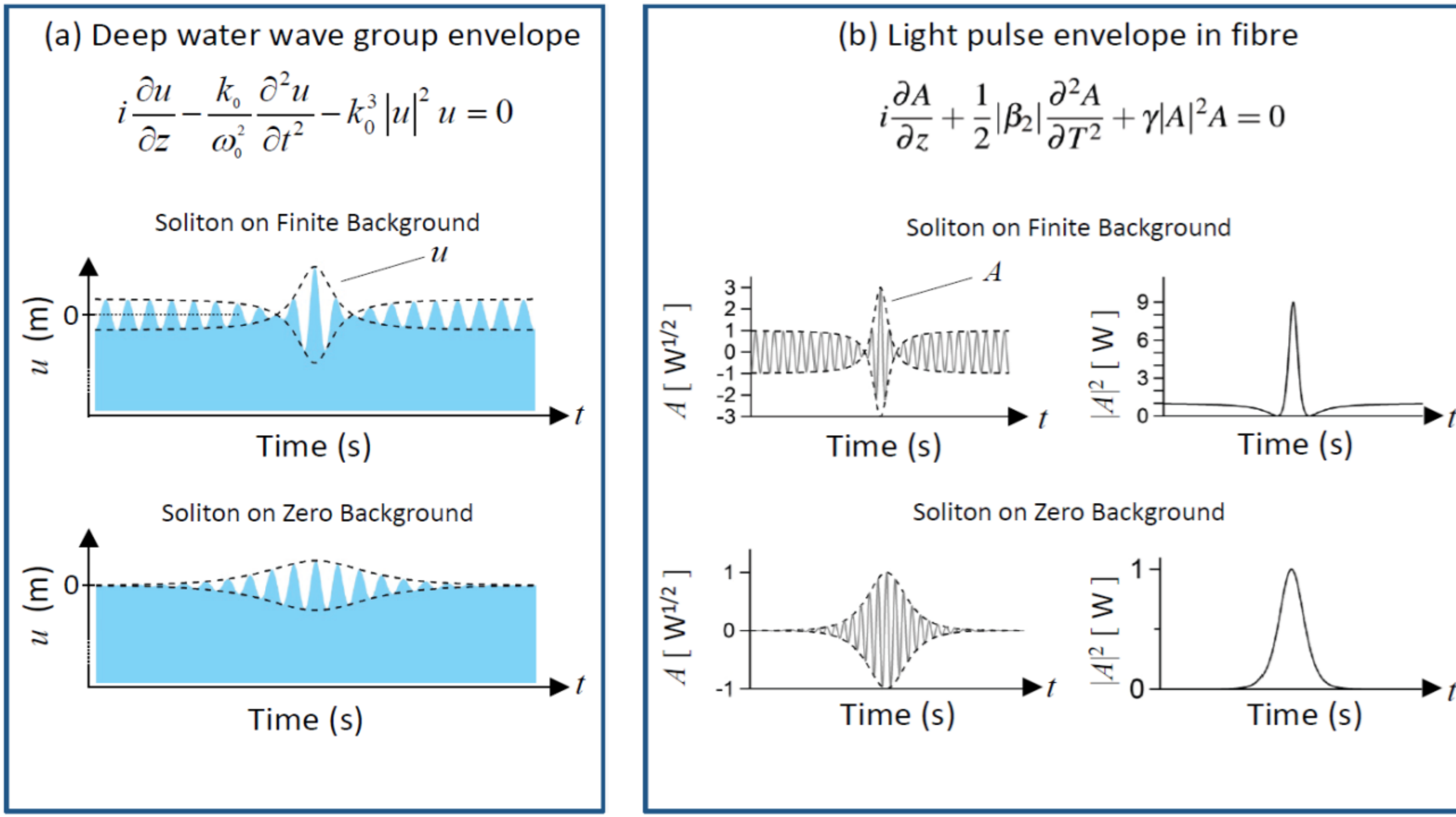


Figure 4.19 The histogram of the significant wave height for the years 1980–2003 for NODC buoy 46005 of Fig. 4.17 (n is the percentage of the total number of occurrences in the interval $\Delta H_s = 0.5$ m).

Envelopes and analogies in the NLSE



This noise emerges from another kind of soliton

One of the most fundamental nonlinear processes in NLSE systems is the exponential growth of a periodic perturbation on a plane wave solution

T. B. Benjamin and J. E. Feir, *J. Fluid Mech.* **27**, 417 (1967)

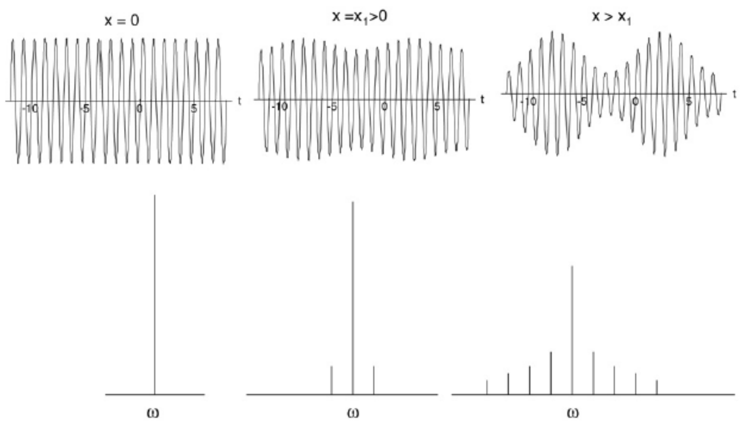
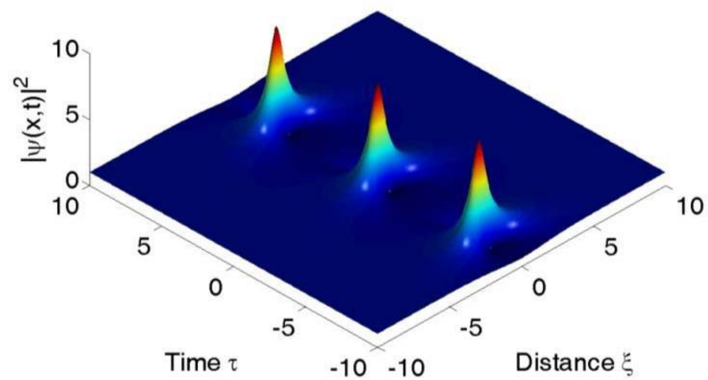


Fig. 1. Top: evolution of a nonlinear wave train in the course of MI. Bottom: the corresponding evolution of wave spectrum.

Solitons on Finite Background

$$i\frac{\partial \psi}{\partial \xi}+\frac{1}{2}\frac{\partial^2 \psi}{\partial \tau^2}+|\psi|^2\psi=0$$

$$\psi(\xi,\tau)=e^{i\xi}\left[\frac{(1-4a)\cosh(b\xi)+ib\sinh(b\xi)+\sqrt{2a}\cos(\Omega\tau)}{\sqrt{2a}\cos(\Omega\tau)-\cosh(b\xi)}\right]$$



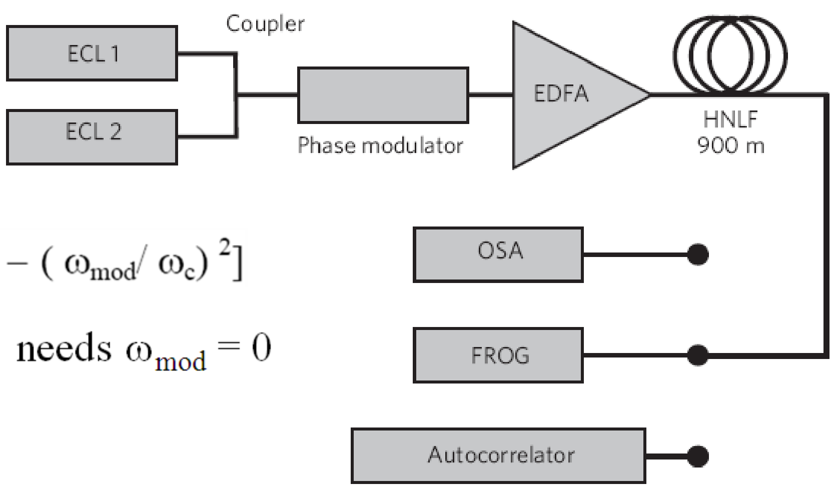
Peregrine, J. Austral. Math. Soc. Ser. B **25**, 16-43 (1983)
Akhmediev & Korneev, Theor. Math. Phys. **69**, 1089-1093 (1986)

Modulation instability - a nonlinear resonance allowing energy transfer from a background to amplify an initial modulation perturbation

Experiments

We can excite families of solitons on finite background using appropriate input conditions into an optical fibre

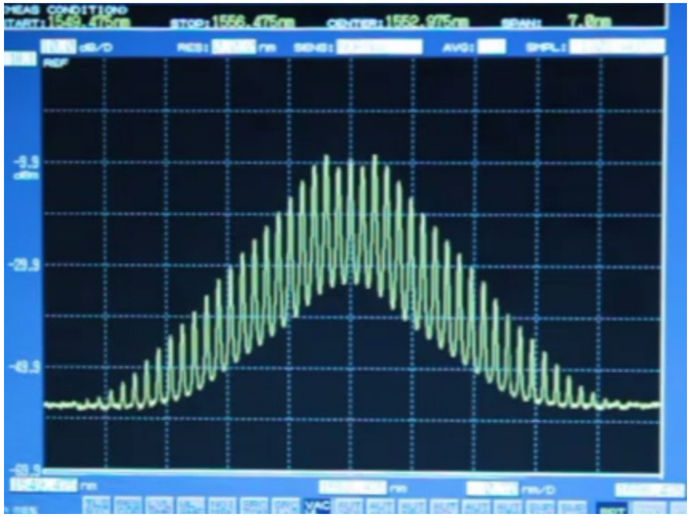
$$i\psi_\xi + \frac{1}{2}\psi_{\tau\tau} + |\psi|^2\psi = 0$$



$$2a = [1 - (\omega_{mod}/\omega_c)^2]$$

$$a = \tfrac{1}{2} \text{ needs } \omega_{mod} = 0$$

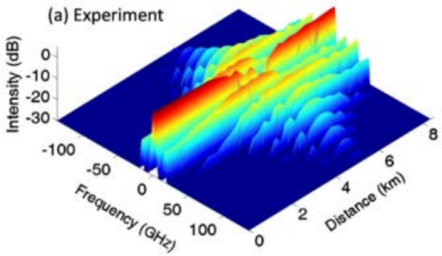
Figure 3 | Experimental set-up. ECL: external-cavity laser; OSA: optical spectrum analyser; FROG: frequency-resolved optical gating. HNLF: highly nonlinear fibre. EDFA: erbium-doped fibre amplifier.

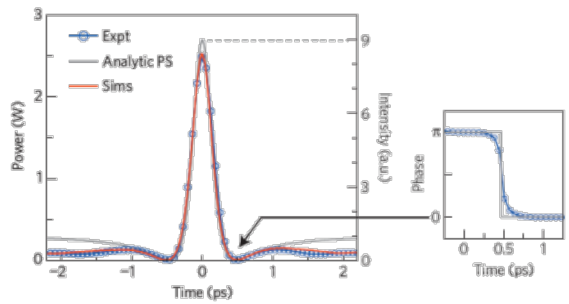


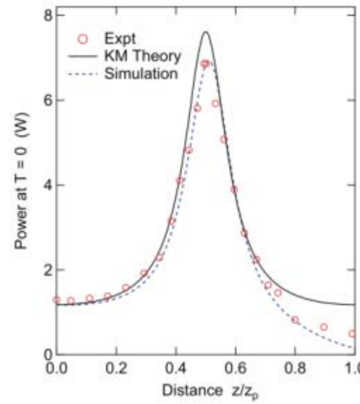
Frequency

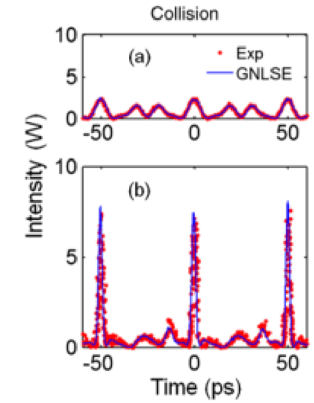
Solitons on Finite Background (SFB)

SFBs have been excited in optics using multi-frequency fields

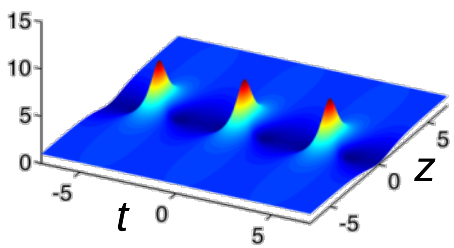


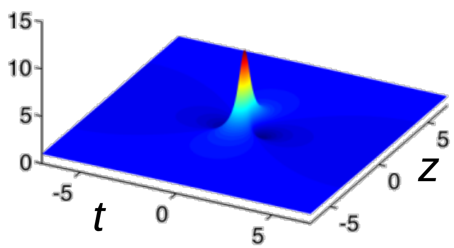


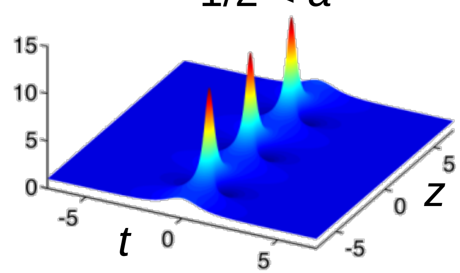


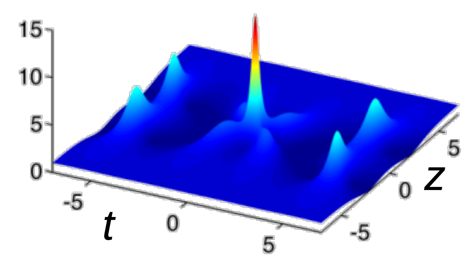


$0 < a < 1/2$	$a = 1/2$	$1/2 < a$	(a, a')

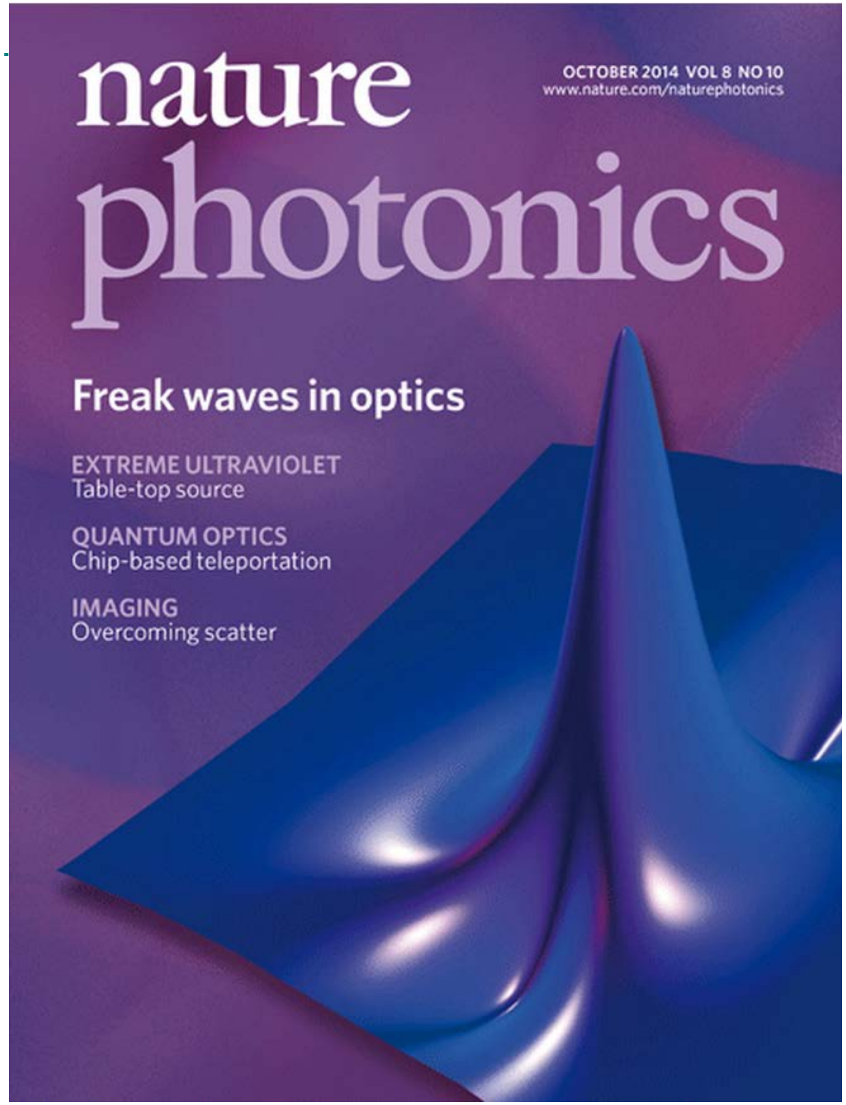








Akhmediev breather (AB)	Peregrine soliton (PS)	Kuznetsov-Ma soliton (KM)	Higher-order AB
Hammani *et al*., OL (2011)	Kibler *et al*., Nat. Phys. (2010)	Kibler *et al*., Sci. Rep. (2012)	Frisquet *et al*., PRX (2013)



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Instabilities, breathers and rogue waves in optics

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Optical rogue waves are rare, extreme fluctuations in the value of an optical field. The term 'optical rogue wave' was first used in the context of an analogy between pulse propagation in an optical fibre and wave group propagation on deep water, but has since been generalized to describe many other processes in optics. This Review provides an overview of the field, concentrating primarily on propagation in optical fibre systems that exhibit nonlinear breather and soliton dynamics, but also discussing other optical systems in which extreme events have been reported. Although statistical features such as long-tailed probability distributions are often considered to be the defining feature of rogue waves, we emphasize the underlying physical processes that drive the appearance of extreme optical structures.

Many physical systems exhibit behaviour associated with the emergence of high-amplitude events that occur with low probability but that have dramatic impact. Perhaps the most widely known examples of such processes are the giant oceanic 'rogue waves' that emerge unexpectedly from the sea with great destructive power[1]. There is general agreement that the physics behind the generation of giant waves is different from that of usual ocean waves, although there is a general consensus that one unique causative mechanism is unlikely. Indeed, oceanic rogue waves have been shown to arise in many different ways: from linear effects such as directional focusing or the random superposition of independent wave trains, to nonlinear effects associated with the growth of surface noise to form localized wave structures[1,2].

The analogous physics of nonlinear wave propagation in optics and hydrodynamics has been known for decades, and the focusing nonlinear Schrödinger equation (NLSE) applies to both systems in certain limits (Box 1). However, the description of instabilities in optics as rogue waves was first used in 2007 when Solli *et al.* reported long-tailed histograms in measurements of intensity fluctuations at long wavelengths in fibre supercontinuum (SC) spectra[3]. An analogy between this optical instability and oceanic rogue waves was suggested for two reasons. First, highly skewed distributions are often considered to define extreme processes, as they predict that high-amplitude events far from the median are still observed with non-negligible probability[4]. Second, the particular regime of SC generation being studied developed from modulation instability (MI), a nonlinear process associated with the exponential amplification of noise that had previously been proposed as a mechanism for generating ocean rogue waves[2].

These pioneering results not only enabled a quantitative analysis of fluctuations at the spectral edge of a broadband SC, but also motivated many subsequent studies into how large-amplitude structures could emerge in optical systems. These studies attracted broad interest and essentially opened up a new field of 'optical rogue wave physics'. Although most research since has focused on wave propagation in optical fibres — particularly in regimes analogous to hydrodynamics — the term 'optical rogue wave' has now been generalized to describe other noisy processes in optics with long-tailed probability distributions, regardless of whether they are observed in systems with a possible oceanic analogy. Moreover, particular analytic solutions of the NLSE that describe solitons on a finite background — often called 'breathers' — are now also widely referred to as rogue wave solutions, even when studied outside a statistical context for mathematical interest, or when generated experimentally from controlled initial conditions. These wider definitions have become well-established, but can unfortunately lead to difficulty for the non-specialist.

Our aim here is to remove any possible confusion by presenting a synthetic review of the field, not in terms of its chronological development, which has been discussed elsewhere[5,6], but rather by classifying rogue waves in terms of their generating physical mechanisms. We begin by discussing rogue waves in the regime of NLSE fibre propagation where MI and breather evolution dominate the dynamics, and then discuss how the effects of perturbations to the NLSE can lead to the emergence of background-free solitons. This provides a natural lead-in to a discussion of the physics and measurement techniques of rogue solitons in fibre SC generation. Finally, we describe the techniques used to control the dynamics of rogue waves in fibre systems, followed by a survey of the results achieved in other systems: lasers and amplifiers, in which dissipative effects are central to the dynamics, and spatial systems, in which both nonlinear and linear dynamics can play a role.

Rogue waves and statistics

Before considering specific examples of optical rogue waves, we first briefly review how rogue wave events are manifested in the statistics of the particular system under study.

In optics, statistical properties are defining features of light sources. For example, the random intensity fluctuations of polarized thermal light follow an exponential probability distribution, and the intensity fluctuations of a laser above threshold follow a Gaussian probability distribution[7]. It was the experimental observation of 'L-shaped' long-tailed distributions[3] that first linked nonlinear optics with the wider theory of extreme events. In a sense, the presence of long-tailed distributions in optics should not be a surprise, as it is well-known that a nonlinear transfer function will modify the probability distribution of an input signal. Indeed, an exponential probability distribution in intensity is transformed under exponential gain to a power-law Pareto distribution. There are other cases, however, in which the functional nonlinear transformation of an input field cannot be identified, because the emergence of high-amplitude events arises from more complex dynamics. Optical rogue waves and long-tailed statistics have been observed in systems that exhibit both types of behaviour.

Rogue waves in optics have been identified in a number of different ways. One approach uses the idea from probability

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Motivating experiments in hydrodynamics

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Rogue Waves: From Nonlinear Schrödinger Breather Solutions to Sea-Keeping Test

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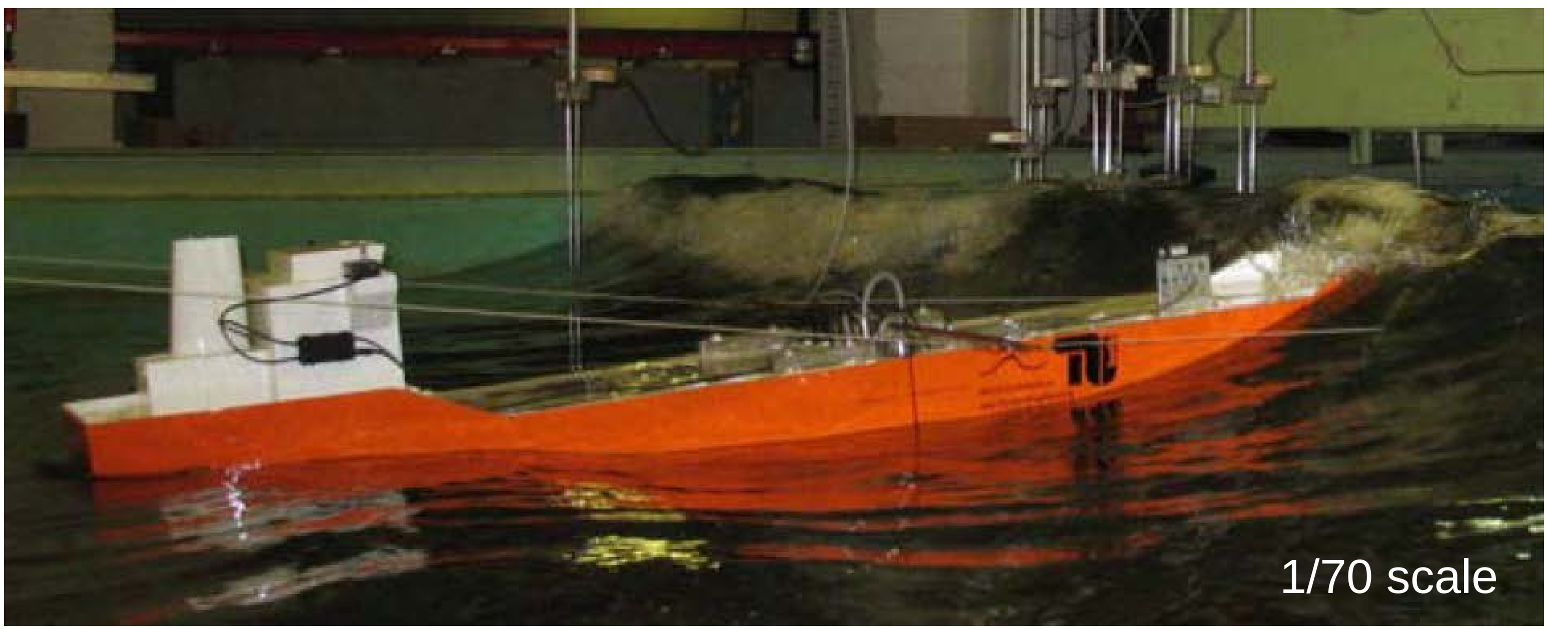
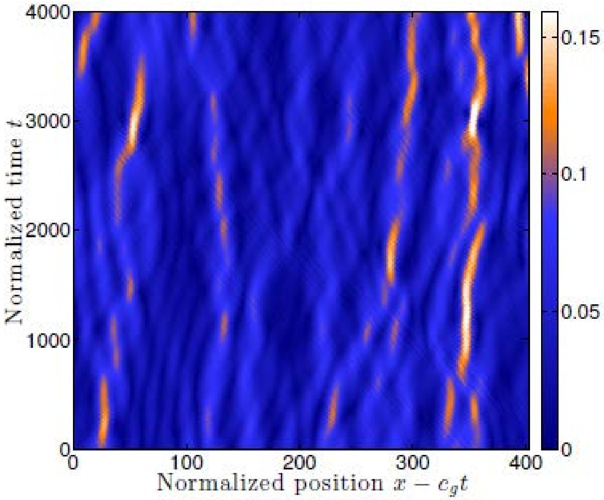


Nonlinear wave enhancement in hydrodynamics

After the first experiments in optics, nonlinear effects are now being studied and observed in a range of hydrodynamic contexts

Extreme wave run up on a vertical cliff
Geophysics Research Letters
DOI: 10.1002/grl.50637 (2013)

Hydrodynamic Supercontinuum
Phys. Rev. Lett. **111**, 054104 (2013)



Hydrodynamics of periodic breathers
Phil. Trans. Roy. Soc. A **372** 20140005 (2014)

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